MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY,

CONTAINING

PAPERS, ABSTRACTS OF PAPERS, AND

REPORTS OF THE PROCEEDINGS

OF THE SOCIETY

FROM NOVEMBER 1918 *TO NOVEMBER* 1919.

VOL. LXXIX.

PUBLISHED BY THE ROYAL ASTRONOMICAL SOCIETY,
BURLINGTON HOUSE, LONDON, W. 1,
AND SOLD BY
WILLIAM WESLEY & SON, 28 ESSEX STREET,
STRAND, LONDON, W.C. 2.
1919.

PRINTED IN GREAT BRITAIN BY NEILL AND CO., LTD., EDINBURGH.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXIX. NOVEMBER 8, 1918. NO. 1

Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT, in the Chair.

The Rev. F. C. Lees, M.A., F.R.G.S., 45 Cavendish Road, Sutton, Surrey;
Hector Munro Macdonald, M.A., F.R.S., Professor of Mathematics, University, Aberdeen;
William John Roberts, Tullyroan, Co. Armagh; and
Captain the Hon. Victor A. Spencer, 50A Curzon Street, London, W. 1,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Henry Allen, Bromham House, near Bedford, Vice-President of the Institution of Mechanical Engineers (proposed by W. H. Maw);
John Edensor Littlewood, F.R.S., Fellow and Lecturer, Trinity College, Cambridge (proposed by A. S. Eddington);
William Evan M'Farlane, M.B., C.H.B., F.R.G.S., Surveyor to Walsh District Hospital, Medical Officer of Health, Irvinebank, North Queensland, Australia (proposed by Ernest H. Beattie);
Lieut.-Commander Edvin Smith, M.A., R.N.V.R., Tillsenburg, Ontario, Canada (proposed by O. T. Olsen);

H. H. Waters, 16 York Avenue, Great Crosby, Liverpool (proposed by F. W. Longbottom); and
Major A. J. Wolff, D.S.O., R.E., Ordnance Survey Office, Southampton (proposed by Sir C. F. Close).

One hundred and nine presents were announced as having been received since the last Meeting, including, amongst others:—

H. Andoyer, Formules et Tables nouvelles relatives à l'étude du mouvement des Comètes (etc.); G. Bigourdan, Observations des Nebuleuses et d'Amas Stellaires (5 volumes); L. Silberstein, Elements of the Electromagnetic Theory of Light; presented by the authors.

H. C. Plummer, Introductory Treatise on Dynamical Astronomy, presented by the Cambridge University Press.

Set of the Proceedings of the London Mathematical Society, first and second series, presented by the Society.

On the Pulsations of a Gaseous Star and the Problem of the Cepheid Variables. Part I. By A. S. Eddington, M.A., F.R.S., Plumian Professor.

1. Although variable stars of the Cepheid type show a periodic change of radial velocity, it is improbable that they are binary stars. The theory which now appears most plausible attributes the light-changes to the pulsation of a single star;* and accordingly the varying radial velocity measures the approach and recession of the surface in the course of the pulsation. In order to throw light, if possible, on the phenomena of these variables, I have investigated the theory of a pulsating mass of gas. A complete solution of this problem would be very difficult; but it seems to be possible to determine the general character of the oscillation, and to obtain results which may be compared with observation.

The type of pulsation here considered is symmetrical about the centre; that is to say, the star remains spherical, but expands and contracts. It is possible that the actual oscillation may be an elliptical deformation; but I think that a symmetrical oscillation is more probable in a star of low density, and it is much simpler to investigate.

It may be useful to summarise some of the leading results of observation with regard to these variables—

(1) The light-curve and the velocity-curve are closely similar. The correspondence is the more marked because both curves are usually very unsymmetrical. Maximum light corresponds to maximum velocity of approach.

* H. Shapley, *Ap. J.*, 40, 448. H. C. Plummer, *Monthly Notices*, 75, 573.

(2) The light-variation is generally marked by a rapid rise to maximum and a slow decline. The velocity-curve shows a corresponding feature, which is usually expressed by saying that the periastron of the "orbit" points directly away from the earth.

(3) The period is a function of the absolute magnitude. For periods from three days upwards, the relation between log-period and absolute magnitude is practically linear; for shorter periods the relation is given by a curve. It appears to be possible to determine the absolute magnitude from the period with a probable error of less than a quarter of a magnitude.

(4) The Cepheids are giant stars, and are much more luminous than the average giants of their type.

(5) The spectral type tends to advance (towards M) as the period increases.

2. In order to gain some general knowledge of these stars and their pulsations, I have obtained the data given in Tables I. and II. for those Cepheids which have been sufficiently investigated. The results in Table II. involve my theory of the radiative equilibrium of the stars*; those in Table I. are independent of that theory.

The orbital data are taken mainly from Ludendorff's annual summary of spectroscopic orbits in the *Vierteljahrsschrift*, and the light-range (when given to hundredths) from a paper by him in *Ast. Nach.*, No. 4625. The mean spectral type, which is of fundamental importance in these calculations, is taken from Shapley, *Ap. J.*, **44**, 274, except for ζ Geminorum, for which a comparable determination by Adams and Joy is used. The type changes during the oscillation, and the arithmetic mean of the highest and lowest values is here adopted. The absolute magnitude is inferred from the period by the use of Shapley's curve in *Pub. Ast. Soc. Pac.*, **30**, 44.† The bolometric magnitude is more suitable for theoretical discussions than the visual, and a small correction has therefore been applied to the visual magnitude of RR Lyræ; the remaining stars being of types F – G require no correction.

From the spectral type we obtain the effective temperature, and hence the radiation per unit area of surface. The absolute bolometric magnitude gives the total radiation. Hence, by division, we obtain the area of the surface. We thus find the radius R. For the relation between type and effective temperature, I have assumed that $\log_{10} T$ changes ·0135 for each step in the spectrum (one-tenth of a type). This gives $A_0 = 11200°$, $F_0 = 8200°$, $G_0 = 6000°$, $K_0 = 4400°$—values which must be fairly near the truth. The resulting values of the radius are, of course, subject

* *Monthly Notices*, **77**, 16, 596.

† Given more conveniently as a table in *Ap. J.*, **48**, 114. This paper was received after my calculations had been made.

TABLE I.

Ref. No.	Star.	Mean Type.	Period (P).	Abs. Mag.	Light Range.	Radius (R).	Semi-Amplitude (δR).	$\frac{\delta R}{R}$.	$\frac{\delta T_e}{T_e}$.	e.	ω.
			d	m	m	km.	km.				°
1	Y Ophiuchi	F9	17·113	−3·99	0·46	42 200 000	1 999 000	·047	·054	0·10	209
2	ζ Geminorum	F2½	10·154	−3·18	0·56	19 400 000	1 798 000	·093	·066	0·22	333
3	S Sagittæ	F8½	8·382	−2·88	0·63	24 600 000	2 000 000	·081	·075	0·35	70
4	W Sagittarii	F5	7·595	−2·72	0·85	18 400 000	1 930 000	·105	·103	0·32	70
5	η Aquilæ	F6½	7·176	−2·64	0·79	19 400 000	1 899 000	·098	·095	0·47	64
6	X Sagittarii	F8	7·012	−2·60	0·67	20 900 000	1 334 000	·064	·080	0·40	94
7	Y Sagittarii	F9	5·773	−2·30	0·74	19 400 000	1 485 000	·077	·089	0·16	32
8	δ Cephei	F6	5·366	−2·19	0·88	15 300 000	1 371 000	·090	·107	0·36	83
9	T Vulpeculæ	F5	4·436	−2·00	0·60	13 200 000	969 000	·073	·072	0·43	111
10	SU Cygni	F1½	3·846	−1·78	0·80	9 570 000	1 350 000	·141	·096	0·21	346
11	RT Aurigæ	F4	3·728	−1·73	0·80	10 900 000	856 000	·078	·096	0·37	95
12	SZ Tauri	F3	3·149	−1·58	0·5	9 580 000	460 000	·048	·059	0·24	77
13	SU Cassiopeiæ	F1½	1·950	−1·17	0·4	7 230 000	295 000	·041	·047	0·0	...
14	RR Lyræ	A5½	0·567	−0·52	0·9	3 700 000	166 500	·045	·109	0·27	97
	Mean	...	...	...	...	...	...	·077	·082	...	...

TABLE II.

Ref. No.	Star	Mass (M). (Sun = 1).	$1-\beta$.	Central Density (ρ_c). gm./cm.^{3}	Central Temperature (T_c). °	T_c/β. $10^6\times$	$P\sqrt{\rho_c}$.	n^2R/g.	a_1. km.
1	Y Ophiuchi	13·0	·614	·0043	3 130 000	8·10	1·12	0·8	45 100 000
2	ζ Geminorum	7·4	·509	·0251	4 920 000	10·0	1·61	0·4	26 400 000
3	S Sagittæ	6·1	·471	·0102	3 460 000	6·53	0·85	1·4	21 800 000
4	W Sagittarii	5·5	·451	·022	4 310 000	7·85	1·12	0·8	19 700 000
5	η Aquilæ	5·2	·440	·0176	3 950 000	7·05	0·95	1·1	18 600 000
6	X Sagittarii	5·1	·435	·0138	3 620 000	6·41	0·82	1·5	18 200 000
7	Y Sagittarii	4·3	·396	·0146	3 520 000	5·83	0·70	2·1	15 100 000
8	δ Cephei	4·0	·382	·028	4 250 000	6·88	0·89	1·3	14 100 000
9	T Vulpeculæ	3·6	·357	·039	4 620 000	7·19	0·88	1·3	12 000 000
10	SU Cygni	3·1	·330	·088	5 710 000	8·52	1·14	0·8	10 300 000
11	RT Aurigæ	3·0	·324	·057	4 880 000	7·22	0·89	1·3	10 000 000
12	SZ Tauri	2·8	·305	·079	5 340 000	7·69	0·88	1·3	8 800 000
13	SU Cassiopeiæ	2·3	·265	·151	6 150 000	8·37	0·76	1·8	6 000 000
14	RR Lyræ	1·7	·197	·833	(9 700 000)	(12·1)	0·52	3·8	2 400 000

TABLE IIA.

(*Spectral Types by Adams and Joy.*)

Ref. No.	Star.	Mean Type.	Radius. km.	$\frac{\delta R}{R}$.	$\frac{\delta T_e}{T}$.	ρ_c.	T_c. °	$P\sqrt{\rho_c}$.	$1-\beta$.
2	ζ Geminorum	F9½	30 000 000	·060	·066	·0068	3 190 000	·84	·509
5	η Aquilæ	G0	24 100 000	·079	·095	·0092	3 170 000	·69	·440
8	δ Cephei	F9½	19 000 000	·072	·107	·0144	3 420 000	·64	·382
11	RT Aurigæ	F8½	14 400 000	·060	·096	·0246	3 690 000	·59	·324
12	SZ Tauri	F8	13 100 000	·035	·059	·0310	3 910 000	·55	·305
	Mean	...	...	·061	·085	...	...	·66	·392

to several sources of uncertainty, but I think they must be substantially correct.

The amplitude δR of the oscillation of radius is the element $a \sin i$ of the supposed orbit. The *major* axis is usually in the line of sight, and consequently the eccentricity does not introduce any correction. In the succeeding column the relative amplitude $\delta R/R$ is tabulated. Perhaps the true values of δR are somewhat greater than those here given, because the spectroscopic measures refer to the integrated light of a hemisphere. If there were no darkening at the limb, and if we make the doubtful assumption that the measures relate to the centre of gravity of the absorption, the rate of change of the star's radius would be $\frac{3}{2}$ of the observed radial velocity. But the limb darkening will eliminate a great part of the oblique surface, and the correction will be smaller.

In so far as the light of the star depends on the temperature, the radiation emitted varies as T^4. It is therefore convenient to take a temperature T_e proportional to the fourth-root of the luminosity, and to tabulate its relative amplitude $\delta T_e/T_e$ in Table I. For the moment, this is best regarded as a conventional way of measuring the light range.

On the pulsation-theory the element e of the orbit is not to be taken literally as an eccentricity; but it gives a measure of the deviation from simple-harmonic oscillation. The longitude of periastron ω should be 90° for a typical Cepheid in which the periastron points away from the earth. This is approximately true in most cases; for Y Ophiuchi, e is small and the discordance of ω is not very important; but SU Cygni (for which the orbit is uncertain) and ζ Geminorum violate the condition. Some of the other tabulated quantities run more smoothly if these two stars are cut out.

Coming to Table II. the mass is deduced by the formulæ in my second paper (*loc. cit.*, p. 604). The mass is simply a function of the absolute magnitude. Similarly the ratio of the force of radiation-pressure to gravitation, $1 - \beta$, is deduced.

From the mass and radius we at once obtain the mean density. I give in the table fifty-four times the mean density, which on my theory gives the central density (for a perfect gas). The central temperature, which should be proportional to $M\beta/R$, is obtained on the same theory.

The remaining columns will be of use later. n is equal to $2\pi/P$, and g is the surface-value of gravity. The last column contains a_1, the semi-axis of the orbit of a hypothetical planet which would revolve round the star of mass already given in the period P.

The determinations of spectral type here used were based mainly on measurements of the hydrogen lines. From a paper by Adams and Joy (*Proc. Nat. Acad., Washington*, 4, 131) it appears that determinations based on the general appearance of the spectrum with higher dispersion give a result on the average four steps lower. I do not know which is likely to be the better

criterion of the effective temperature.* For comparison I give in Table IIA. the chief results for five of the stars for which determinations by Adams and Joy are available. The masses are the same as in Table II.

3. We notice first † that the maximum (central) density is less than 0·1, except for SU Cassiopeiæ and RR Lyræ, and consequently the stars are in a truly gaseous condition throughout. For RR Lyræ, and perhaps SU Cassiopeiæ, the deviations from the laws of a perfect gas will be important near the centre. RR Lyræ is typical of a large class of short-period Cepheids (cluster-variables), and we must expect to find the behaviour of these somewhat different from that of the truly gaseous stars in the table. It is probably significant that the linear relation between log period and absolute magnitude holds for the truly gaseous stars, but changes to a curve when the period is less than three days, this being the point at which the central density begins to be too high.

The relative amplitudes of the pulsations, $\delta R/R$, are of much the same magnitude in the different stars, and are roughly equal to the temperature amplitudes $\delta T_e/T_e$. The light-ranges are usually not well determined, and $\delta T_e/T_e$ is probably subject to considerable accidental error. There is some indication that the two amplitudes are strictly proportional, and perhaps equal, especially if we omit ζ Geminorum, SU Cygni, and RR Lyræ for the reasons already given. The average amplitude is about one-twelfth, and probably the maximum possible is about one-ninth; the absence of smaller amplitudes may be due to selection.

The table also gives striking evidence against the binary hypothesis. Although the line of argument has been given already by Shapley,‡ the objections are perhaps made plainer when illustrated by actual numbers. We see that the radius (δR) of the supposed orbit of the principal star is on the average one-twelfth of the radius (R) of the star.§ This would only be possible if the secondary had a very small mass, which can therefore be neglected in comparison with the mass of the primary. Thus, knowing the mass and period of the system, we can calculate the mean distance between the centres of the two components. This is tabulated under a_1 in Table II. Comparing R and a_1, we see that in most cases the secondary is just inside the primary; and, since the eccentricity is large, it will dip further into the

* We are here only concerned with the *mean* type and the *mean* rate of radiation during the pulsation; and it is assumed that the relation between them should be the same as for a non-vibrating star. The relation between their fluctuations is not likely to be so direct, so I make no use of the measured range of spectral type in this paper.

† Unless otherwise stated, Tables I. and II. are referred to. In general, the results of Table IIA. are equally favourable to the conclusions stated.

‡ *Ap. J.*, 40, 448.

§ In particular cases, $\sin i$ might be small and the radius of the orbit much larger than here stated. But this could scarcely happen for all fourteen stars.

primary at periastron. The binary interpretation thus seems to be inadmissible.

It will also be noticed that $P\sqrt{\rho_c}$ (Table II.) is approximately constant, the range (again omitting Nos. 2, 10, 14) being 0·70 to 1·12. On the binary hypothesis it seems incredible that the period should have anything to do with the density of the primary, whereas a relation of this kind is predicted by almost any pulsatory theory.

4. In the course of the pulsation, the volume of the star is in many cases nearly doubled, and the consequent changes of temperature in the interior must be prodigious. It is at least worth investigating whether such a vibration could continue. Would not its energy be at once dissipated by the flow of heat tending to level the wave of temperature?

The equations for the oscillations of a gas, taking into account the flow of heat, lead to a differential equation of the fourth order, which can only be treated by quadratures. There are four boundary conditions to bring in, two at the centre and two at the boundary. Moreover, the quadrature near the boundary becomes impracticable owing to the vanishing of the density and pressure. In these circumstances there was a difficulty in finding a starting-point for attacking the problem.

This difficulty disappeared when it was realised that owing to the high opacity inside the star the vibrations near the centre must be very approximately adiabatic. In a former paper I have shown that more than half the radiation is absorbed in a layer $1\frac{1}{2}$ metres thick at atmospheric density. The explanation is that the high-temperature radiation within the stars resembles X-rays rather than ordinary light, and is absorbed on a correspondingly higher scale. This opacity tends to conserve the distribution of heat in the long waves here considered.

We start then by considering the adiabatic oscillations of a gas. We can afterwards calculate the flow of heat which would result, and determine the point in the star at which it becomes so great as to render our approximation invalid. Beyond this point the conditions would have to be treated by a different method.

5. *Adiabatic Oscillations of a Star.*

Let p, ρ, T be the pressure, density, and temperature at a point distant ξ from the centre; and let g be the acceleration of gravity at this point. We fix attention on a particular element of matter, so that ξ oscillates in the course of a pulsation. Let p_0, ρ_0, T_0, ξ_0, g_0 be the values in the undisturbed state, and δp, $\delta\rho$, etc., be the deviations from the undisturbed values.

We write

$$\delta p/p_0 = p_1,$$

or

$$p = p_0(1+p_1) \qquad \rho = \rho_0(1+\rho_1) \qquad \xi = \xi_0(1+\xi_1) \quad . \quad (1)$$

and similarly for all the other variables. If the period of the oscillation is $2\pi/n$, p_1, ρ_1 . . . will contain a factor $\cos nt$. We

consider a small oscillation, and neglect the square of the amplitude.

For adiabatic oscillations the pressure and density of a *particular element* * are connected by

$$p = k\rho^{\gamma} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where γ is the ratio of specific heats, so that

$$\frac{\delta p}{p_0} = \gamma \frac{\delta \rho}{\rho_0},$$

i.e.

$$p_1 = \gamma \rho_1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The matter which is in the spherical shell ξ to $\xi + d\xi$ occupies in the undisturbed state the shell ξ_0 to $\xi_0 + d\xi_0$. Hence, equating the mass,

$$\rho \xi^2 d\xi = \rho_0 \xi_0^2 d\xi_0 \quad . \quad . \quad . \quad . \quad (4)$$

Differentiating, this gives

$$\frac{\delta \rho}{\rho_0} + \frac{2\delta\xi}{\xi_0} + \frac{d\delta\xi}{d\xi_0} = 0.$$

Hence by (1)

$$\rho_1 = -2\xi_1 - \frac{d(\xi_0 \xi_1)}{d\xi_0} = -3\xi_1 - \xi_0 \frac{d\xi_1}{d\xi_0} \quad . \quad . \quad (5)$$

The ordinary hydrodynamical equation is

$$\frac{1}{\rho}\frac{dp}{d\xi} = -g - \frac{d^2\xi}{dt^2} \quad . \quad . \quad . \quad . \quad (6)$$

$$= -g + n^2 \xi_0 \xi_1.$$

Hence, using (4)

$$\frac{1}{\rho_0 \xi_0^2}\frac{dp}{d\xi_0} = -\frac{g}{\xi^2} + \frac{n^2 \xi_0}{\xi^2}\xi_1 \quad . \quad . \quad . \quad (7)$$

Now $g/\xi^2 = M/\xi^4$ where M is the mass interior to ξ, which remains the same as the star oscillates. Hence

$$\delta\left(\frac{g}{\xi^2}\right) = \delta\left(\frac{M}{\xi^4}\right) = -\frac{4M}{\xi_0^5}\delta\xi = -\frac{4g_0}{\xi_0^2}\xi_1,$$

so that

$$\frac{g}{\xi^2} = \frac{g_0}{\xi_0^2} - \frac{4g_0}{\xi_0^2}\xi_1.$$

Hence (7) gives

$$\frac{1}{\rho_0 \xi_0^2}\frac{d}{d\xi_0}(p_0 + p_0 p_1) = -\frac{g_0}{\xi_0^2} + \frac{4g_0}{\xi_0^2}\xi_1 + \frac{n^2}{\xi_0}\xi_1,$$

which, if we recall that p_1 and ξ_1 contain a factor $\cos nt$, separates into the equilibrium formula

$$\frac{dp_0}{d\xi_0} = -g_0\rho_0 \quad . \quad . \quad . \quad . \quad (8)$$

* The undisturbed state is not supposed to be that of adiabatic equilibrium.

and the formula for the variation from equilibrium values

$$\frac{d(p_0 p_1)}{d\xi_0} = \rho_0(4g_0 + n^2\xi_0)\xi_1 \quad . \quad . \quad (9)$$

Using (8), (9) becomes

$$p_0\frac{dp_1}{d\xi_0} - g_0\rho_0 p_1 = (4g_0 + n^2\xi_0)\rho_0\xi_1 \quad . \quad . \quad (10)$$

But from (3) and (5)

$$p_1 = -\gamma\left(3\xi_1 + \xi_0\frac{d\xi_1}{d\xi_0}\right) \quad . \quad . \quad . \quad (11)$$

Eliminating p_1 from (10) and (11), we have finally,

$$\frac{d^2\xi_1}{d\xi_0^2} + \frac{4-\mu}{\xi_0}\frac{d\xi_1}{d\xi_0} + \left(\frac{n^2\rho_0}{\gamma p_0} - \left(3 - \frac{4}{\gamma}\right)\frac{\mu}{\xi_0^2}\right)\xi_1 = 0 \quad . \quad (12)$$

where $\mu = g_0\rho_0\xi_0/p_0$.

6. Equation (12) has to be solved numerically. We first form an auxiliary table of the quantities μ and $u(=p_0/\rho_0)$, which occur in the coefficients. These depend on the distribution of density inside the star, and I take this from my theory of radiative equilibrium given in the papers already quoted. The law of density corresponds to one of the laws discussed by Emden, and in Table III. the column u is taken directly from his calculations, *Gaskugeln*, p. 85.

The column ξ gives the distance from the centre, the unit being very nearly R/6·9. u represents p_0/ρ_0, or the temperature, the value at the centre being the unit. μ is of the dimensions of a pure number; it is easily shown to be equal to $-4\xi du/ud\xi$. The column M, taken from *Gaskugeln*, p. 80, shows the mass interior to the distance ξ; it is of interest as showing how small a proportion of the mass is in the outer layers.

TABLE III.

ξ.	u.	μ.	M.	ξ.	u.	μ.	M.
0	1·00000	·00000	0	3¼	·31553	6·8256	...
¼	·98969	·08266	·005	3½	·27625	7·5439	1·820
½	·95985	·32260	·039	3¾	·24096	8·3227	...
¾	·91355	·69825	·120	4	·20926	9·1887	1·920
1	·85507	1·17948	·252	4¼	·18075	10·176	...
1¼	·78899	1·73433	·428	4½	·15505	11·334	1·974
1½	·71951	2·33426	·630	4¾	·13183	12·729	...
1¾	·64999	2·95732	·841	5	·11079	14·465	2·001
2	·58285	3·58921	1·045	5½	·07426	19·727	...
2¼	·51962	4·22279	...	6	·04371	38·743	2·016
2½	·46112	4·85722	1·399	6½	·01784	69·639	...
2¾	·40766	5·49634	...	6·8	·00414	287·02	2·016
3	·35922	6·14848	1·655	6·8999	·00000	∞	2·016

In equation (12) the factor $\frac{n^2}{\gamma}\frac{\rho_0}{p_0}$ may be written

$$\frac{1}{u}\frac{n^2}{\gamma}\left(\frac{\rho_0}{p_0}\right)_c,$$

the suffix c indicating values at the centre. If we write

$$\omega^2 = \frac{n^2}{\gamma}\left(\frac{\rho_0}{p_0}\right)_c \quad . \quad . \quad . \quad . \quad (13)$$

and

$$\alpha = 3 - 4/\gamma \quad . \quad . \quad . \quad . \quad (14)$$

equation (12) becomes

$$\xi_1'' + \frac{4-\mu}{\xi_0}\xi_1' + \left(\frac{\omega^2}{u} - \frac{\alpha\mu}{\xi_0^2}\right)\xi_1 = 0 \quad . \quad . \quad . \quad (15)$$

Here the only uncertain factor is α, which depends on the ratio of specific heats. Its greatest possible value (for a gaseous star) is 0·6, corresponding to $\gamma = \frac{5}{3}$; this is only attained if the whole internal energy of the star is the translational motion of the molecules. The minimum value of α is 0, corresponding to $\gamma = \frac{4}{3}$; it is well known that for lower values of γ a spherical mass of perfect gas is unstable.

As an example I have taken $\alpha = 0{\cdot}2$. It is then necessary to try various values of ω^2, *i.e.* try various periods, until we find the value which gives a wave that fits the star and so determines the natural period of oscillation. For the fundamental oscillation, the first node (or place of constant pressure and density) will fall at the boundary of the star.

In Table IV. solutions for $\omega^2 = {\cdot}055$, ·060, ·065 respectively have been calculated. We start from the centre with an arbitrary value of ξ_1 (according to the required amplitude of the pulsation) here taken as unity. It is clear from the differential equation that ξ_1' must be zero initially. The difficulty arising from the indeterminate form of ξ_1'/ξ_0 at the centre is got over by obtaining an approximate solution in series valid as far as $\xi_0 = 1$. From that point the solution is continued by quadratures, using Table III. I have used Darwin's formulæ for the quadratures (*Collected Papers*, 4, 16); in the outer parts it was necessary to retain differences up to a high order.

Considering the solution for $\omega^2 = {\cdot}060$, we see that ξ_1'' has become negative at $\xi_0 = 5$, and is decreasing very rapidly. Hence ξ_1' has begun to diminish, and will become negative before the boundary is reached. The node which is given by the vanishing of $3\xi_1 + \xi_0\xi_1'$ (equation (11)) will fall within the star. The wave is thus a little too short. If we take $\omega^2 = {\cdot}065$, ξ_1'' becomes negative at $\xi_0 = 3\frac{3}{4}$, and the wave is very much shorter. The large change shows that ·060 must be very near the true solution, and a value of ω^2 very little smaller than this will fit the conditions. Taking $\omega^2 = {\cdot}055$ greatly overshoots the mark, ξ_1'' never becomes negative, but goes to infinity at an increasing rate, so that the wave is much too long.

TABLE IV.

$\left(\alpha=0{\cdot}2\ ;\ \gamma=\frac{10}{7}\right)$.

	$\omega^2=\cdot055$.			$\omega^2=\cdot060$.			$\omega^2=\cdot065$.		
ξ_0.	ξ_1.	ξ_1'.	ξ_1''.	ξ_1.	ξ_1'.	ξ_1''.	ξ_1.	ξ_1'.	ξ_1''.
0	1	0	·0423	1	0	·0413	1	0	·0403
1	1·0218	·0443	·0504	1·0212	·0431	·0476	1·0206	·0420	·0448
1¼	1·0345	·0573	·0538	1·0335	·0554	·0506	1·0325	·0535	·0474
1½	1·0505	·0713	·0585	1·0489	·0685	·0541	1·0474	·0657	·0497
1¾	1·0702	·0867	·0644	1·0678	·0825	·0584	1·0654	·0784	·0524
2	1·0940	·1037	·0718	1·0903	·0977	·0634	1·0867	·0919	·0550
2¼	1·1223	·1227	·0806	1·1168	·1142	·0688	1·1114	·1059	·0570
2½	1·1556	·1441	·0912	1·1475	·1320	·0744	1·1396	·1202	·0577
2¾	1·1946	·1685	·1041	1·1829	·1514	·0804	1·1715	·1346	·0568
3	1·2401	·1965	·1203	1·2234	·1723	·0862	1·2069	·1484	·0529
3¼	1·2932	·2290	·1407	1·2692	·1945	·0917	1·2456	·1606	·0440
3½	1·3551	·2672	·1676	1·3208	·2181	·0968	1·2870	·1697	·0276
3¾	1·4272	·3135	·2045	1·3784	·2427	·0999	1·3300	·1735	−·0014
4	1·5122	·3707	·2571	1·4422	·2678	·0994	...	...	...
4¼	1·6131	·4442	·3361	1·5122	·2919	·0927	...	...	...
4½	1·7349	·5427	·4621	1·5879	·3130	·0735	...	...	...
4¾	...	...	...	1·6680	·3266	·0289	...	...	...
5	...	...	...	1·7497	·3233	−·0680	...	...	...

Our adiabatic approximation will be found to break down near the boundary, so that we cannot formulate the exact conditions. But this scarcely matters. So long as the node falls well inside the star (where the approximation holds) we evidently have to go on lengthening the wave; and when we have overshot the mark a little the wave gets quite out of control even in the region where our approximation is valid. Thus the true solution must be very near $\omega^2=\cdot060$, however the outside conditions are modified.

It is not difficult to see from the differential equation that the required value of ω^2 will be nearly proportional to α, especially when α is small. I have found for $\alpha=0{\cdot}6$, $\omega^2=\cdot156$, so that the proportion holds fairly well up to the extreme limit. For a further check, Mr H. E. Green has calculated the solution for $\alpha=0{\cdot}1$, obtaining $\omega^2=\cdot0315$; this solution is accurately comparable with $\alpha=0{\cdot}2$, $\omega^2=\cdot060$, since in both cases ξ_1'' became negative at $\xi_0=5$. I therefore adopt as sufficiently accurate

$$\omega^2=\tfrac{3}{10}\alpha \qquad . \quad . \quad . \quad . \quad (16)$$

We shall use this result in § 8 to calculate the theoretical period of pulsation of a star.

It should be noticed that the actual boundary condition for a

free oscillation is not $p_1 = 0$, but $p_0 p_1 = 0$. Dividing (9) by (8), we have

$$\frac{d(p_0 p_1)}{dp_0} = -\left(4 + \frac{n^2 \xi_0}{g_0}\right)\xi_1 \quad . \quad . \quad . \quad (17)$$

Since p_0 and $p_0 p_1$ vanish at the boundary, we have in the limit

$$p_1 = -\left(4 + \frac{n^2 \xi_0}{g_0}\right)\xi_1 \cdot \quad . \quad . \quad . \quad (17a)$$

at the boundary. It will be seen from Table II. that in the actual Cepheids $n^2 R/g_0$ is usually about unity. Hence near the boundary p_1 is equal to about $-5\xi_1$.

7. *The Ratio of Specific Heats.*—Hitherto we have not referred explicitly to the radiation pressure, which plays an important part especially in stars so massive as the Cepheids. It will be seen, however, that our analysis is correct if by p we understand the whole pressure, *i.e.* the gaseous pressure + radiation pressure. In that case we must examine more closely what is the significance of the constant γ which may be regarded as defined by equation (2), or

$$\frac{\delta p}{p_0} = \gamma \frac{\delta \rho}{\rho_0}.$$

Æthereal energy behaves as though the æther had a ratio of specific heats equal to $\frac{4}{3}$—just like a molecular medium whose energy is half translational (kinetic) and half internal (potential). We may expect then that in a mixture of æthereal and material energy the effective ratio of specific heats will be a weighted mean between $\frac{4}{3}$ and the value for the matter alone. This rule is fairly accurate; but it is perhaps worth while to obtain the exact expression.

The radiant energy per unit volume at temperature T is

$$aT^4$$

where a is the constant $7{\cdot}06 \cdot 10^{-15}$. The radiation-pressure is

$$\tfrac{1}{3}aT^4.$$

Thus the total energy per unit volume E and the total pressure p are given by

$$E = c_v \rho T + aT^4 \quad . \quad . \quad . \quad . \quad (18)$$

$$p = \mathfrak{R}\rho T + \tfrac{1}{3}aT^4 \quad . \quad . \quad . \quad . \quad (19)$$

If Γ is the ratio of specific heats for the matter alone,

$$\mathfrak{R} = c_p - c_v = (\Gamma - 1)c_v.$$

If V is the volume of unit mass, the adiabatic condition is

$$\delta(VE) + p\delta V = 0,$$

which gives by (18) and (19), remembering that $V\rho$ remains constant,

$$V(c_v\rho + 4aT^3)\delta T + aT^4\delta V + \{(\Gamma - 1)c_v\rho T + \tfrac{1}{3}aT^4\}\delta V = 0.$$

Hence

$$\frac{\delta T}{T} = -\frac{\delta V}{V}\frac{(\Gamma - 1)c_v\rho T + \frac{4}{3}aT^4}{c_v\rho T + 4aT^4} \quad . \quad . \quad . \quad (20)$$

or

$$T_1 = \rho_1 \frac{(\Gamma - 1) + \frac{4}{3}\epsilon}{1 + 4\epsilon} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where $\epsilon = aT^4/c_v\rho T$ = ratio of æthereal to material energy . (22)

Again

$$\begin{aligned} V\delta p &= \delta(Vp) - p\delta V \\ &= \delta(V(p + E)) \\ &= V(\Gamma c_v\rho + \tfrac{16}{3}aT^3)\delta T + \tfrac{4}{3}aT^4\delta V \end{aligned}$$

$$= \delta V \left\{ \tfrac{4}{3}aT^4 - (\Gamma c_v\rho T + \tfrac{16}{3}aT^4)\frac{(\Gamma - 1) + \frac{4}{3}\epsilon}{1 + 4\epsilon} \right\} \quad \text{by (20).}$$

Hence

$$\frac{\delta p}{p} = \frac{\delta V}{V}\left\{ \frac{\frac{4}{3}\epsilon(1 + 4\epsilon) - (\Gamma + \frac{16}{3}\epsilon)(\Gamma - 1 + \frac{4}{3}\epsilon)}{(\Gamma - 1 + \frac{1}{3}\epsilon)(1 + 4\epsilon)} \right\} .$$

The factor in the bracket gives the required value of γ, since by definition $\delta p/p = \gamma\delta\rho/\rho = -\gamma\delta V/V$. It can be reduced to the more convenient form

$$\frac{\gamma - \frac{4}{3}}{\Gamma - \frac{4}{3}} = \frac{\Gamma - 1 + \frac{4}{3}\epsilon}{\Gamma - 1 + \frac{1}{3}\epsilon} \cdot \frac{1}{1 + 4\epsilon} \quad . \quad . \quad . \quad (23)$$

By (22) we have

$$\begin{aligned} \frac{\epsilon}{\Gamma - 1} &= \frac{aT^4}{\Re\rho T} \\ &= \frac{3(1 - \beta)}{\beta} \end{aligned}$$

where $(1 - \beta)$ is the ratio of the radiation-pressure to the whole pressure, or the ratio of the force of radiation-pressure to gravitation.

Hence

$$\frac{\gamma - \frac{4}{3}}{\Gamma - \frac{4}{3}} = \frac{4 - 3\beta}{1 + 12(\Gamma - 1)(1 - \beta)/\beta} \quad . \quad . \quad . \quad (24)$$

From this we can tabulate the values of γ corresponding to different values of Γ and $1 - \beta$. The latter quantity depends only on the mass or absolute magnitude of the star, and has been given for the chief Cepheids in Table II. Γ is at present unknown, but presumably it will be nearly the same for all stars.

TABLE V.

	Values of γ.			Values of $(\gamma\alpha)^{\frac{1}{3}}$.		
$1-\beta$. \ Γ =	$1\frac{4}{3}$.	$1\frac{5}{3}$.	$1\frac{6}{3}$.	$1\frac{4}{3}$.	$1\frac{5}{3}$.	$1\frac{6}{3}$.
·20	1·410	1·467	1·511	·478	·632	·730
·30	1·398	1·443	1·476	·439	·573	·655
·35	1·392	1·433	1·462	·421	·546	·621
·40	1·387	1·423	1·449	·401	·519	·589
·45	1·382	1·414	1·437	·382	·493	·558
·50	1·377	1·406	1·426	·363	·466	·527
·60	1·368	1·390	1·405	·321	·412	·464
·70	1·359	1·375	1·386	·277	·353	·397
·80	1·350	1·361	1·368	·225	·286	·321

8. *Period of the Pulsation.*—In equation (13) Emden's unit of length, R/6·9, has been employed. In order to make the equation independent of the units chosen, we must write it

$$\omega^2 = \frac{n^2}{\gamma}\left(\frac{\rho_0}{p_0}\right)_c\left(\frac{\mathrm{R}}{6\cdot9}\right)^2 \quad . \quad . \quad . \quad . \quad (25)$$

which makes the dimensions consistent, ω^2 being a pure number.

Since $n = 2\pi/\mathrm{P}$, and we have found $\omega^2 = \frac{3}{10}\alpha$, this gives

$$\mathrm{P}\sqrt{\rho_c} = \mathrm{C}(\gamma\alpha)^{-\frac{1}{2}} \quad . \quad . \quad . \quad . \quad (26)$$

where

$$\mathrm{C} = \frac{2\pi\rho_c\mathrm{R}}{6\cdot9}\sqrt{\frac{10}{3p_c}} \quad . \quad . \quad . \quad . \quad (27)$$

Now, from equation (43) in my former paper (*Monthly Notices*, **77**, 601) it appears that p_c is proportional to $\rho_c{}^2\mathrm{R}^2$ in all stars. (It must be remembered that in the present paper p is the whole pressure, whereas in the paper quoted p is the gaseous pressure, which is smaller in the ratio β.) Consequently C is an absolute constant. To determine its value we may use the standard star discussed in my former papers (*loc. cit.*, p. 22), for which

$$\mathrm{R} = 7 \times 10^{11} \text{ cm.}$$
$$\rho_c = \cdot1085 \text{ gm./cm.}^3$$
$$p_c = 2\cdot535 \times 10^{13} \text{ dynes/cm.}^2$$

which gives C = 25080 in C.G.S. units.

Or, more conveniently, if the day is the unit of time,

$$\mathrm{P}\sqrt{\rho_c} = 0\cdot290\ (\gamma\alpha)^{-\frac{1}{2}} \quad . \quad . \quad . \quad (28)$$

The factor $(\gamma\alpha)^{-\frac{1}{2}}$ will vary a little from one Cepheid to another, since it depends on $1-\beta$, *i.e.* on the mass or absolute magnitude. But we see from Table V. that the variation due to this will be comparatively trifling. Hence $\mathrm{P}\sqrt{\rho_c}$ will be nearly constant, and this is confirmed by observation as already noticed (Table II.). This relation has already been pointed out by Shapley; and we

must not take it as confirming the details of our theory, because it is a dimensional relation which would follow on almost any hypothesis of pulsation under gravity.

If all the Cepheids were perfectly homologous, $P\sqrt{\rho_c}$ would be rigorously constant. In our theory the factor $(\gamma\alpha)^{-\frac{1}{2}}$ represents a change in the elastic constants as the mass of the star changes. This arises, not from any change in the properties of the material, but because the energy tends to pass more and more from the matter to the æther as the mass increases. Owing to observational errors, we can scarcely hope to test this suggested progression at present.

A more precise test of the theory is afforded by the predicted value of the constant C. Taking the mean for the nine typical Cepheids in Table II. (omitting Nos. 1, 2, 10, 13, 14), we have $1-\beta=0{\cdot}40$, and $P\sqrt{\rho_c}=0{\cdot}89$. By Table V. this gives $(\gamma\alpha)^{\frac{1}{2}}=\cdot401$ $\cdot519$, $\cdot589$, according as we assume $\Gamma=1\frac{4}{9}$, $1\frac{5}{9}$, $1\frac{2}{3}$; and the corresponding values of C from equation (26) are then

$$0{\cdot}35,\ 0{\cdot}46,\ 0{\cdot}52.$$

This is a remarkable verification of the predicted value 0·29, seeing that we might well have arrived at a result ten- or a hundredfold out if the theory were erroneous. The agreement would be exact if $\Gamma=1{\cdot}40$.

The above values of $1-\beta$ and $P\sqrt{\rho_c}$ for the mean of nine stars are practically identical with those for δ Cephei; we may therefore consider δ Cephei more particularly as representing the average result of the table. Its period is $5^d{\cdot}37$. From the spectral type and absolute magnitude the theory predicts by equation (28) that its period will not be less than $2^d{\cdot}88$. This corresponds to $\Gamma=\frac{5}{3}$. Actually there must be some energy of ionisation and internal atomic energy which will reduce Γ and make the period somewhat longer; but it is very improbable that the period should be excessively longer. It is true that if Γ were extremely close to $\frac{4}{3}$ the period would be much longer; but this value of Γ has no greater probability than other values, so that a sufficiently close approach could only happen by an unlikely coincidence.

Knowing nothing as to the value of Γ, except that it is a physical constant lying between $\frac{4}{3}$ and $\frac{4}{3}+\frac{1}{3}$ with no obvious preference for any part of this range, the chances are 9 to 1 that it lies in the range $\frac{4}{3}+\frac{1}{30}$ to $\frac{4}{3}+\frac{1}{3}$. Taking the latter limits, and ignoring the unfavourable $\frac{1}{10}$ chance, the theory then fixes the period of δ Cephei as between

$$7{\cdot}22 \text{ and } 2{\cdot}88 \text{ days.}$$

If Adams' and Joy's determination of its type is used instead of Shapley's, the limits are

$$10{\cdot}01 \text{ and } 4{\cdot}01 \text{ days.}$$

In either case the observed period $5{\cdot}37$ days is in good agreement.

If we use the spectral types of Adams and Joy and take the mean for the five stars in Table IIa., the constant C is found to be

$$0{\cdot}27,\ 0{\cdot}35,\ 0{\cdot}39,$$

according as $\Gamma = 1\frac{4}{9},\ 1\frac{5}{9},\ 1\frac{2}{3}$, so that the theoretical value 0·29 agrees with observation if $\Gamma = 1{\cdot}47$. Moreover, Table IIa. shows that there is actually a progressive increase of $P\sqrt{\rho_c}$ with $1-\beta$, which roughly agrees with the change of $(\gamma a)^{-\frac{1}{2}}$ predicted by theory; but I think this must be partly a matter of luck, as I can scarcely believe the observational data trustworthy to this degree.

We may conclude that the theory predicts the period from a knowledge of the type and absolute magnitude with an uncertainty (due to ignorance of Γ) which may be estimated as represented by the factor 2; and that the observed periods, both in the mean and individually, are found to agree easily within this limit.

9. *The Flow of Heat.*—We have yet to verify that adiabatic oscillation is a valid approximation throughout the greater part of the interior.

If the net outward flow of radiation per unit area per unit time is $\frac{4}{3}\pi B$, we have (equation (35), p. 598, of my paper quoted),

$$B = -\frac{\mu}{k\rho}\frac{dT^4}{d\xi} = -\frac{ac}{4\pi k\rho}\frac{dT^4}{d\xi},$$

where c is the velocity of light, and k the mass-coefficient of absorption.

The outward flow of heat across a sphere of surface $4\pi\xi^2$ is thus,

$$\begin{aligned} H &= -\tfrac{1}{3}\pi\frac{ac}{k\rho}\xi^2\frac{dT^4}{d\xi} \\ &= -\tfrac{4}{3}\pi\frac{ac}{k\rho_0\xi_0^{\,2}}\left(\xi^4\frac{dT^4}{d\xi_0}\right) . \qquad . \qquad . \qquad (29) \end{aligned}$$

The variation of $\xi^4\dfrac{dT^4}{d\xi_0}$ is

$$4\left(\xi_0^{\,4}\frac{dT_0^{\,4}}{d\xi_0}\xi_1 + \xi_0^{\,4}\frac{d}{d\xi_0}(T_0^{\,4}T_1)\right),$$

which, since $T_0^{\,4}/p_0$ is constant throughout the star, and $T_1 = \dfrac{\gamma-1}{\gamma}p_1$ in adiabatic motion,*

$$\begin{aligned} &= 4\xi_0^{\,4}\frac{T_0^{\,4}}{p_0}\left(\xi_1\frac{dp_0}{d\xi_0} + \frac{\gamma-1}{\gamma}\frac{d}{d\xi_0}(p_0p_1)\right). \\ &= 4\xi_0^{\,4}\frac{T_0^{\,4}}{p_0}\frac{\rho_0}{\gamma}\{(\gamma-1)n^2\xi_0 + (3\gamma-4)g_0\}\xi_1 \quad \text{by (8) and (9)} \quad (30) \end{aligned}$$

* This is not quite rigorous when γ is the effective ratio of specific heats for a mixture of material and ætherial energy, *cf.* equation (21); but the error is not important.

Substituting in (29) this gives the excess outward flow of heat across the sphere

$$H_0H_1 = -\tfrac{16}{3}\pi\frac{T_0^4}{p_0}\frac{ac}{k\gamma}\xi_0^2\xi_1\{(\gamma-1)n^2\xi_0+(3\gamma-4)g_0\} \quad . \quad (31)$$

The gain of heat by a spherical shell ξ_0 to $\xi_0+d\xi_0$ is thus per unit volume

$$\frac{dQ}{dt} = -\frac{1}{4\pi\xi_0^2}\frac{d}{d\xi_0}(H_0H_1) = \tfrac{1}{3}\frac{aT_0^4}{p_0}\frac{4c}{k\gamma}\frac{1}{\xi_0^2}\frac{d}{d\xi_0}[\xi_0^2\xi_1\{(\gamma-1)n^2\xi_0+(3\gamma-4)g_0\}]$$

$$= (1-\beta)\frac{4c}{k\gamma}\frac{1}{\xi_0^2}\frac{d}{d\xi_0}[\xi_0^2\xi_1\{(\gamma-1)n^2\xi_0+(3\gamma-4)g_0\}] \quad (32$$

$$= (1-\beta)\frac{4c}{k\gamma}\{-n^2T_1+(3\gamma-4)\frac{1}{\xi_0^2}\frac{d}{d\xi_0}(g_0\xi_0^2\xi_1)\} \quad . \quad (33$$

since

$$\rho_1 = -\frac{1}{\xi_0^2}\frac{d}{d\xi_0}(\xi_0^3\xi_1) \quad \text{by (5).}$$

Suppose this gain of heat, hitherto neglected in our equations, produces a temperature oscillation of amplitude T_0T_2. Consider only the first term in (33) then, the gain is

$$\frac{4c}{k\gamma}(1-\beta)n^2T_1 = c\rho_0\frac{d}{dt}(T_0T_2)$$

$$= c\rho_0 in T_0T_2,$$

where c is the specific heat, and the factor i indicates that the phases of T_1 and T_2 differ by a quarter-period.

In order to obtain the order of magnitude of T_2 we can neglect the unimportant factors $(1-\beta)$ and γ and put $c=\mathfrak{R}$, thus obtaining

$$iT_2 = \frac{4c}{k}\frac{n}{p}T_1 \quad \text{roughly.}$$

Take a Cepheid of period about eight days so that $n=10^{-5}$; then $c=3\cdot10^{10}$, and I have found elsewhere that $k=24$, thus

$$iT_2 = \frac{50000}{p}T_1.$$

So long as the pressure is not less than 10 atmospheres (10^7 dynes per sq. cm.) T_2 is less than 1/200 of T_1. This pressure is reached at a comparatively small depth below the photosphere; and through the whole region interior to it the temperature wave will be practically that given by T_1, as hitherto assumed, the leakage of heat represented by the wave T_2 being insignificant in comparison. As we approach the centre where p is of order 10^{14} the adiabatic approximation becomes far more exact.

The second term in (33) can be treated similarly. It is of the same order of magnitude. Our assumption of adiabatic motion is

thus justified, the error being probably no greater than in the ordinary applications of the theory of sound.

10. *Change of Period.*—We have verified from the observational data the approximate relation that the period varies inversely as the square-root of the density. Theoretically the result holds strictly for stars of constant mass, since the factor $(\gamma\alpha)$ varies a little with the mass. We can scarcely doubt that in the case of a slight change of density of a particular star, due to its evolutionary contraction during the period it has been under observation, the relation must be still more closely fulfilled. This should be true even if the type of pulsation differs from that here investigated.

The period of a Cepheid can be determined with great accuracy; it is not unusual to give it to seven significant figures in the ephemerides. It would be of great interest to determine the change of period (if any) of these stars, some of which have been under observation for many years; because this would give a means of measuring a very slight change of density, and so determine the rate of stellar evolution and the length of life of a star.

Taking δ Cephei as typical, I find for it—

$$\text{Exhaustion of potential energy} = 3{\cdot}7 \times 10^{48} \text{ ergs.}$$
$$\text{Annual radiation} = 1{\cdot}0 \times 10^{44} \text{ ergs.}$$

Neither quantity can be subject to much uncertainty. The former is equal to $\frac{3}{2}M^2/R$, and it is only in respect of the particular factor $\frac{3}{2}$ that the theory of radiative equilibrium has been invoked. The annual radiation is obtained directly from the absolute magnitude, which is fairly well determined. In order to provide the energy radiated, the potential energy must decrease annually by 1 part in 37,000; actually not more than $\frac{1}{3}$ of this energy is available for radiation (the rest being used in raising the temperature of the star) so that the decrease must be at least 1 in 12,000. It follows that the radius must decrease annually by 1 in 12,000, the density increase by 1 in 4000, and the period decrease by 1 in 8000, or about 40 seconds annually. This is on the usual hypothesis that the heat is provided by the energy of contraction.

I think that such a change of period is far too great to have escaped observation. Chandler considered it certain that the period of δ Cephei is decreasing $0^s{\cdot}05$ annually. I scarcely know what reliance should be placed on this as a positive determination, but it evidently negatives the possibility of the large change here suggested. This seems to be strong evidence that the heat radiated by a star cannot be provided by contraction.

If Chandler's value of the annual decrease $0^s{\cdot}05$ is correct, the annual change of period is 1 in 9,300,000, and of density 1 in 4,600,000. δ Cephei would take 3,200,000 years to double the density, or about ten million years to pass from type G to type F. Remembering that progress in the dwarf stage is much

slower than in the giant stage—owing to the smaller radiation, and to the condition that the star is drawing on, instead of storing up, internal heat,—this would probably square with the more recent estimates of the age of terrestrial rocks, and the time-scale required for many evolutionary processes. It would help to settle a most important and controversial question if consistent results could be obtained from a number of Cepheids.

11. The relations already found are theoretically sufficient to determine completely the physical state of a Cepheid if one property, say the absolute magnitude, is given. For example—

Take a star of absolute bolometric magnitude $-5^{m}\cdot00$. According to the theory of radiative equilibrium its mass will be 27·6, and $1-\beta=\cdot725$. Using Adams' and Joy's determinations of spectral types, we found $\Gamma=1\cdot47$; hence, by (24), $(\gamma a)^{\frac{1}{2}}=\cdot296$.

The visual magnitude will be slightly different from the bolometric, and we have to guess a suitable value, afterwards modifying it by trial and error. I find that visual magnitude $-4^{m}\cdot90$ will fit. Then Shapley's empirical curve gives the period P = 30·8 days.

By (28) we have

$$30\cdot8\sqrt{\rho_c}=0\cdot29/0\cdot296,$$

so that the central density is ·00101, and the mean density is $\frac{1}{54}$ of this.

From the mass and density we deduce the radius

$$R=87{,}800{,}000 \text{ kilometres.}$$

We now know the area of the surface, and combining this with the absolute bolometric magnitude, we calculate the effective temperature

$$T_e=5420^{\circ}.$$

This corresponds to type $G3\frac{1}{2}$ according to the scale adopted in § 2. For this temperature the correction visual – bolometric magnitude is $+0^{m}\cdot1$ (Table III., p. 605, *loc. cit.*), so that our assumed visual magnitude is verified.

From the mass, density, and β we derive, according to the theory of radiative equilibrium, a central temperature

$$T_c=2{,}300{,}000^{\circ}.$$

In this calculation we have pressed the theory to an extreme degree. Our object is not so much to assert the truth of the conclusions, as to use every opportunity of discovering by comparison with observation the directions in which our approximate treatment may be improved.

For comparison I give the results for Cepheids of magnitudes $-5^{m}\cdot00$ and $-2^{m}\cdot00$ respectively. The last column corresponds to a hypothetical emendation of the luminosity-period relation, which is considered below.

TABLE VI.

Period.	30·8 days.	4·5 days.	*30·8 days.*
Absolute visual magnitude	$-4^m\cdot9$	$-2^m\cdot0$	*$-4^m\cdot3$*
Absolute bolometric magnitude	$-5^m\cdot0$	$-2^m\cdot0$	*$-4^m\cdot4$*
Mass (sun = 1)	27·6	3·6	*17·4*
$1-\beta$	·725	·357	*·661*
Radius	87 800 000 km.	16 400 000 km.	*79 500 000 km.*
Central density	·0010	·0204	*·00086*
Central temperature	2 300 000°	3 750 000°	*1 970 000°*
Effective temperature	5420°	6290°	*4960°*
Spectral type	$G3\frac{1}{2}$	$F8\frac{1}{2}$	*G6*

In these calculations we have used only the empirical law connecting luminosity and period, together with the constant $\Gamma = 1\cdot47$ derived from the mean of a number of observations. The rest is pure theory.

On most of these quantities we have no observational check, but the spectral types can be observed. It will be seen that we predict that the longer period corresponds to a later (redder) spectral type, and this is in accordance with observation; but it is open to doubt whether the agreement is quantitatively correct.

According to some mean results collected by Shapley a change of period from four to thirty days should give a change of type nearly from F to M; the data are uncertain and subject to revision. But, taking his more accurate determinations in *Ap. J.*, 44, 274, I find—

5	stars.	Mean period	2·96	days.	Mean type F2·9
3	,,	,,	5·19	,,	,, F6·7
7	,,	,,	7·58	,,	,, F7·8
2	,,	,,	22·1	,,	,, F8·8

which, so far as it goes, confirms the slower change given by the theory.*

It must be noted that our calculation depends on Shapley's luminosity-period curve. The upper part of this curve appears to depend entirely on observations of photographic magnitude which have been reduced to visual magnitude by applying the colour correction which is here in dispute. If the colour correction for the long-period stars is decreased (as we suggest), the luminosity-period curve will be altered, and our calculations must be correspondingly corrected. It will be found that the theoretical change of type with period is increased; that is to say, if any concession can be made in the direction we require, the theory will advance a little to meet it.

* Shapley's types are systematically higher than Adams' and Joy's, which formed the basis of the foregoing calculation, but this does not affect the differential results here discussed.

In the last column of Table VI. the luminosity-period relation is amended as here suggested. The star being of type G6 would have a colour-index about 1·0 instead of 1·6 assumed by Shapley; accordingly, we have changed its absolute visual magnitude from −4·9 to −4·3 to correspond. The progression of type between periods of four and thirty days is raised 50 per cent. by this change.

The point must be left unsettled until more observations of the long-period Cepheids are obtained. Meanwhile it seems desirable to state the unbiassed prediction of the approximate theory.

So far we have dealt chiefly with the theoretical determination of the length of period. In Part II. we shall discuss the rate of decay of the oscillation, the limit to the possible amplitude, the secondary terms which give the characteristic shape to the velocity-curve and light-curve, as well as certain other points on which the mathematical theory is less explicit.

On the Radiative Equilibrium of the Stars. A Correction.
By A. S. Eddington, M.A., F.R.S., Plumian Professor.

In former papers on this subject (*Monthly Notices*, 77, pp. 16, 596,) I have taken the value of the radiation-pressure to be $\frac{1}{3}aT^4$, where aT^4 is the density of the radiant energy. The correct value is $\frac{1}{3}aT^4$, and the force of radiation-pressure on an element is accordingly $d(\frac{1}{3}aT^4)$.

The error unfortunately runs through a great number of formulæ.* In particular the principal results, (41) and (42), become when corrected—

$$\frac{p}{\frac{1}{3}aT^4} = \frac{\beta}{1-\beta} \quad . \quad . \quad . \quad . \quad . \quad (41a)$$

$$T_1^4 = \frac{4}{k_0 a} g_1 (1-\beta) \quad . \quad . \quad . \quad . \quad (42a)$$

But the numerical results are not much affected, and the error can be compensated by multiplying the adopted molecular weight by $\sqrt{2}$. The results in the second paper, §§ 15–19, hold for molecular weight 2·8 instead of 2; this is perhaps an advantage, since 2 was chosen as the extreme lower limit, and a value near 3 is more likely to represent the actual conditions. The corrected value of the opacity is 21·6, instead of 5·4.

In § 9, however, an important difference is made in my calculation of the heat available for radiation. The proportion available

* A somewhat simplified investigation of the formulæ, in which this error is corrected, will appear shortly in the *Astrophysical Journal*.

is much less than I supposed. If Ω is the negative potential energy, we now have

$$\text{internal ætherial energy} = (1-\beta)\,\Omega$$

$$\text{internal material energy} = \frac{\beta}{3(\gamma-1)}\Omega,$$

so that when $\gamma = \frac{4}{3}$, the total heat in the star is equal to the energy of contraction, and it is only when $\gamma > \frac{4}{3}$ that there is any balance for radiation. It has long been known that this criterion applies when ætherial energy is neglected; and the revised calculation now shows that the presence of ætherial energy makes no difference. The reason is that the æther behaves as though it had a ratio of specific heats $\frac{4}{3}$; consequently the ratio of specific heats for the star is effectively brought nearer to $\frac{4}{3}$, but an excess cannot be changed into a defect or *vice versa*.

With regard to my calculations of the rate of evolution of a giant star on the contraction-hypothesis (§ 20), the reduced proportion of heat available for radiation will make the evolution even faster.

Note on the Nebulosity round Nova Persei. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. We may hope soon to hear of attempts to photograph the neighbourhood of Nova Aquilæ No. 3, for detection of possible surrounding nebulosity. Since the remarkable photographs of Nova Persei in 1901–2, the power of available instruments has been considerably increased, which justifies enhanced expectations. Mr. Perrine has recently reminded astronomers of the desirability of testing the light for polarisation. No sensible polarisation was detected in the Nova Persei nebula, but this does not necessarily negative the view that the light received by us was reflected light —the "light-echo" of the maximum flare of the Nova, for the reflecting matter may not have been finely enough divided to show such effects. The spectroscopic evidence, which indicated that the spectrum of the reflected light agreed with that of the sudden flare, and did not correspond with that which the star gave subsequently, is good support of the view that the sudden flare was responsible for the gradually extended illumination of a surrounding nebulous mass. In the present note this hypothesis will be adopted. Treating the flare as momentary, the illuminated surface at any time T subsequently is an ellipsoid of revolution, with foci at the Nova and the observer. If the distance from Nova to observer be D, the major axis of the ellipsoid is $D + VT$, where V is the velocity of light. Since D is large, we may regard the ellipsoid as sensibly a paraboloid, with focus S at the Nova, and parameter proportional to T. As the paraboloid accordingly expands

it intersects any surface existing in the nebula in a changing section, which gives an appearance of movement.

2. Without some further hypothesis, expressed or understood, nothing can be inferred as to the shape or dimensions of the nebula. Certain estimates made originally were either mistaken, or rested on some assumption which might not be true, as, for instance, that the light was travelling at right angles to the line of sight. What follows is also based on assumptions which may not be true; but they will at least be clearly stated. The reasons for making them are indicated in *M.N.*, 72, p. 405, and are connected with a suggested origin for Novæ: viz. that a diffused *non-rotating* nebula may fall together under its own gravitation, the final stages being very rapid and culminating in the luminous outburst observed.

3. It is an essential feature of this suggestion that the nebula should have no sensible rotation, so that it would be disposed in concentric spherical layers. The Owl nebula and some of the globular clusters may be cited in support of the view that rotation, plainly present in many nebulæ, may nevertheless in other instances be insensible. Let us first examine the conditions for contraction simultaneously as a whole. Consider a particle originally at distance r_0 from the centre. The spherical shell outside it has no attraction upon it, while the spherical mass within, of mass m_0, say, attracts it as though concentrated at the centre; and under this force the particle will arrive at the centre in a time which varies as m_0/r_0^3. If this time is the same for all the particles of the nebula, m_0 must vary as r_0^3; that is, the sphere must originally be homogeneous. Under these circumstances it will remain homogeneous, the density becoming greater and greater until we get the luminous outburst accompanied by something of the nature of an explosion which rescatters the concentrated matter.

4. Such a hypothesis would account for the sudden blazing out of a nova, but would not explain the accompanying existence of a surrounding nebula, such as was revealed to us in the case of Nova Persei. According to the hypothesis of homogeneity, all the nebula should have contracted together. Suppose, however, that we vary the hypothesis in one or other of two ways, supposing the density greater towards the centre or towards the boundary. First consider a central condensation. As an illustration let us take for a moment our solar system with a considerable mass at the centre. Suppose the planets, arrested in their orbits at the same instant, to fall in towards the sun. The nearer ones would arrive long before the others, the times of falling being similar to their periods of revolution. Mercury would have fallen into the sun, and caused a "flare up," before Neptune had much more than started. It is easy to see on these lines how, in the case of a non-rotating nebula with central condensation, the inner parts might fall in, to cause a nova by congestion, while the outer parts were still diffused. [If, further, the outer parts of the nebula were rotating we might have an additional reason for their being retained in

extension. The pictures of the Andromeda nebula suggest an outer extension which is rotating, with a central portion in which the effects of rotation are less obvious, and in the centre of which Nova Andromedæ appeared. This possibility of the existence of rotating and non-rotating portions in the same nebula seems worth considering. In the solar system we have the rotating part represented by the planets, while the comets give a rough representation of the non-rotating portions moving in elongated orbits which are an approximation to falls into the sun. But this idea does not immediately concern us.]

5. Suppose now that the departure from homogeneity is in the other direction, the matter being concentrated not at the centre, but in a shell away from the centre: and as an extreme case suppose the shell empty inside. Then a particle on the interior surface of the shell has no attraction to the centre, and remains for the time at rest: but a particle on the outside of the shell feels the attraction of the whole shell and begins to move inwards. The outside of the shell becomes the inside and loses the attraction, while what was originally the inside, being now outside, begins to be attracted. There would thus be a tendency for the shell to remain a shell by alternations between its surfaces, accompanied by a general contraction. [We shall be led in what follows to contemplate the possibility of two such shells coexisting in the same nebula.] To sum up, the effect of central condensation would be to separate the parts of the nebula, drawing the inner parts in more quickly and leaving the outer ones behind; while that of shell condensation would be to keep the shell together so long as the particles composing it could move freely through one another.

6. Let us now see how far such ideas are consistent with the facts observed in the case of Nova Persei, as shown in Ritchey's beautiful drawings in *Ap. J.*, 15 (1902), plates vi. to x. We shall study especially the two rings shown in all these plates, and chiefly their approximate dimensions, which we may take to be on the extreme dates as follows:—

Date.	Days elapsed since "Flare."	Ring Diameter. Inner.	Outer.
1901 Sept. 20	209	14′	25′
1902 Feb. 8	350	15	30

These rings are the intersections of surfaces (within or bounding the nebula) with the paraboloids for the two dates, and it is suggested that the surfaces are separate spherical surfaces with the Nova as centre. This was not the supposition first made, and was only arrived at after considering various other possibilities. At first it was supposed that the rings both belonged to a single surface: and when this supposition would not work, that one at least of the surfaces was lenticular or very oblate, like the rings of the Andromeda nebula and some other nebulæ. But instead of

following the history of these trials throughout, it will be simpler to start with the conclusions in mind.

7. It will be noticed that the ratio of the "days elapsed" is sensibly as 5 to 3. Let us draw (as a section of the surfaces of revolution round the view-axis) two parabolas (fig. 1), with parameters $3k$ and $5k$, the equations of which, taking SX and S$B$$b$ as the axes of x and y, will be

$$y^2 = 12kx + 36k^2$$
$$y^2 = 20kx + 100k^2.$$

If either of the rings is due to a spherical surface with centre S, cutting these parabolas in W and V, then, since SV = SW, and

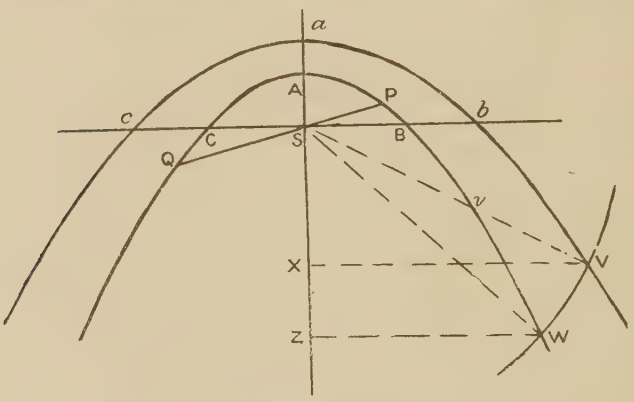


Fig. 1.

since these focal distances are equal to the distances of V and W from the directrices, we see that

$$XZ = \text{dist. between directrices} = 4k.$$

If the surface WV is prolate (while still having the Nova at S as centre), the distance XZ will be greater than $4k$; if oblate, it will be less than $4k$. Now prolate spheroids are not within our experience of nebulæ. It seems fair to assume that the surfaces must be either oblate or spherical, so that XZ cannot exceed $4k$. This gives us a limit to the value of k. For let us substitute the observed radii of the outer ring in the equations, taking 1′ of arc as the unit of length. We have

$$(12'5)^2 = 156'3 = 12kx_1 + 36k^2$$
$$(15'0)^2 = 225'0 = 20kx_2 + 100k^2.$$

Thus

$$\frac{x_1 - x_2}{4k} = \frac{0'445}{k^2} + \frac{1}{2},$$

and if this ratio is not to exceed unity, k must exceed 0′·94.

8. Dealing similarly with the inner ring of light, presuming that it also belongs to a shell round the Nova as centre, we get

$$\frac{x_1 - x_2}{4k} = \frac{0''.32}{k^2} + \frac{1}{2},$$

which gives the limit $0''.80$ as a lower one for k. Since this is less than $0''.94$, there does not at first sight appear to be any difficulty in satisfying both conditions by making k exceed $0''.94$; and by making the excess larger we can introduce more oblateness into the surfaces. But a new consideration now arises. The value of x_2 is

$$x_2 = 2''.81/k - 5k,$$

which becomes negative when $k = 0''.75$; and as we further increase k, the point V on the outer parabola rises above SBb, the axis of y. The legs SW and SV are now on opposite sides of this axis, and it becomes more difficult to satisfy the condition that the ring shall increase in size, *i.e.* that y_2 (for the second date) shall be greater than y_1 (for the first date). The measures of diameter of the rings are subject to much uncertainty, and it is not intended to claim that they are accurate. There may even be a slight decrease in size at the second date, but it cannot be large.

9. Let us in the first instance assume the measures accurate, so that $y_2 > y_1$. As k increases there comes a value when we can no longer pass any surface concave to the centre S through the two positions of the ring. For let the equation to such a surface be

$$Ax^2 + By^2 = 1,$$

so that if $(x_1 y_1)$ and $(x_2 y_2)$ both lie on it,

$$Ax_1^2 + By_1^2 = 1$$
$$Ax_2^2 + By_2^2 = 1.$$

The ratio of B to A is thus

$$\frac{B}{A} = \frac{x_1^2 - x_2^2}{y_2^2 - y_1^2}.$$

If then $y_2 > y_1$ and B and A are both positive, we must have $x_1^2 > x_2^2$. The limiting case is given by $x_2 + x_1 = 0$ when $B = 0$. The y-axis of the surface becomes infinite, and it degenerates (as an extreme case of oblateness) into two parallel planes with S midway between them. The corresponding value of k is deducible from the condition $x_1 + x_2 = 0$ when x_1 and x_2 are substituted in the two parabolas, so that

$$20y_1^2 + 12y_2^2 = 1920k^2.$$

Since y_1 and y_2 are nearly equal, if y be their mean value we have

$$y^2 = 60k^2,$$

and putting

$$y = 7''.0, \quad k = 0''.91,$$
$$y = 7''.5, \quad k = 0''.97.$$

10. If k be greater than this, B becomes negative, and the surface surrounding S is a hyperboloid. We thus have a superior limit for k which is very close to the inferior limit indicated by the outer ring. The precise value of the superior limit depends on the precise radii of the inner ring, which are subject to uncertainty, as remarked, but it cannot be far from the value $k = 0''.94$.

11. Suppose now we admit as possible a slight *decrease* in the inner ring from the first date to the second, so that $y_2 < y_1$. Then we must have $x_1^2 < x_2^2$ if B and A are to be both positive. The upper limit for k, deduced from the consideration of increase in the ring, is now removed; but we must re-examine the condition of prolateness. If the surface is not to be prolate, we must have

$$B < A,$$

therefore

$$(x_2^2 - x_1^2) < (y_1^2 - y_2^2).$$

We may therefore make x_2^2 rather greater than x_1^2, the excess being limited by the amount of decrease we can admit in the inner ring, which cannot be large. When this limit is reached the surface becomes a sphere, and beyond the limit (*i.e.* when the value of k is further increased) it becomes prolate. A simple case will show how quickly we approach unworkable conditions. Taking the radius of the inner ring at the first date to be $7''.0$, let us assume that the ring lies on a sphere of radius $5k$. Now this sphere touches the second parabola at the vertex, so that at the second date the ring would have shrunk to a point. To fit the facts we must expand the ring by making the surface considerably prolate. The value of k thus ruled out is approximately $1''.4$.

12. Summing up, if y be the mean value of the radius of the inner ring for the two extreme dates selected, then k cannot much exceed the value given by

$$y^2 = 60k^2,$$

or say,

$$k = 0''.94.$$

If we assign values to the radii showing a slight increase from first date to second, then the limit for k arises from this increase itself. If, on the other hand, we consider that there is a slight decrease, the limit arises from the necessity of keeping the surrounding surface oblate or spherical.

13. The value $k = 0''.94$ is thus assigned with considerable precision. We cannot have a *greater* value of k without straining the measures of the inner ring, or making them refer to a prolate surface; we cannot have a *less* value without making the outer ring belong to a prolate surface. We can fit all the measures by putting $k = 0''.94$ and making both surfaces spherical, in which case we have, say, for the inner ring,

$$x_1 = -x_2 = 1''.88, \quad y_1 = y_2 = 7''.28;$$

and for the outer ring,

$$x_1 = 11''.2, \quad x_2 = 7''.2, \quad y_1 = 12''.6, \quad y_2 = 15''.0.$$

14. This value of k gives us, of course, the scale of the picture in light years, and therefore the distance of the Nova. Light travels from S to a and back again, a distance of $10k$ or 9″·4 in 350 days. The travel per day is thus 0″·0269, and from Nova to observer it is

$$\frac{57{\cdot}3 \times 60}{{\cdot}0269 \times 365} = 350 \text{ years},$$

which dates the outburst in A.D. 1551. Though a date not far from this was assigned on the hasty supposition that the light was travelling perpendicular to the line of sight, it must be remembered that this supposition would give very different distances according as we used the September or February date, the traverses being nearly equal, while the times elapsed are very different.

15. The parallax of the Nova corresponding to the above distance in light years is 0″·0093. Direct measurements by Bergstrand (*A.N.*, No. 3834) assigned a parallax of 0″·03 ±0″·009, but whether they suffice to rule out the smaller parallax above deduced is open to question. The Greenwich results (*M.N.*, 62, p. 489) indicated a parallax "probably considerably less than 0″·1."

16. There are two difficulties in accepting the hypothesis of spheres concentric with the Nova which have not hitherto been noticed—

(*a*) That the Nova seems to be distinctly excentric in several of the pictures.

(*b*) That the inner ring seems in some of the pictures to be elliptic rather than circular, being elongated in the direction transverse to that of the above apparent excentricity.

17. Let us take the second difficulty first. It can be met by assuming that the spherical surfaces have lateral extensions which give the appearance of elongation, especially to the inner ring. We are familiar with the appearance of two arms, springing from opposite parts of the nucleus, in the case of the Andromeda and other spiral nebulæ. They are believed to originate under tidal forces, and their spiral form is associated with the rotation of the nucleus. We have been led to contemplate the existence of spherical surfaces with no rotation, but they may still be subject to tidal forces, giving rise to lateral extensions. Reference to Ritchey's drawings (*Ap. J.*, **15** (1902), plates vi. to x.) will give the arguments for and against this view. It is possibly significant that the extensions seem to lie near the plane of the Galaxy.

18. The apparent excentricity of the Nova lies in the direction nearly perpendicular to the Galaxy, and is so pronounced, to a casual inspection of plate vi., for instance, or again in the earlier drawings (plates xviii. and xix. of vol. **14**, 1901), that it was originally taken as the keynote of the situation. A start was made with the idea of a very oblate spheroid, with axis slightly inclined to the view-axis XS. An extreme case of oblateness would be a

flat disc: and if we imagine PSQ (fig. 1) to be the section of such a disc, cutting the first parabola in P and Q, then it is easily seen that S will be excentric in PQ, while at the same time the boundary of the disc need not depart far from the circular form. Its axis perpendicular to the paper will be nearly equal to BC, and PQ is not much larger than BC for small inclinations. In fact, if the inclination be θ, then

$$SP = \frac{12k}{1+\sin\theta} \qquad SQ = \frac{12k}{1-\sin\theta},$$

so that the diameter PSQ is of length $24k \sec^2\theta$, the increase being only of the second order in θ, while the excentricity is of the first order. But the hypothesis of a flat disc requires that the rings should expand at a wholly prohibitive rate. The precise rate of expansion is much affected by uncertainties of measurement; but the very roughest measures show that it cannot be of anything like the magnitude required, which would be in the ratio of the elapsed times, viz. 3 to 5. In order to fit the observed rates of expansion it was found necessary to assume a spheroidal surface, while still retaining sufficient oblateness to meet the demands of the apparent excentricity of the Nova: for with a spherical surface there can, of course, be no inclination of the axis. The various trials made revealed a fundamental difficulty in satisfying all the observations, which was ultimately identified as above.

19. If we accept the above view that the surfaces are spherical, except in so far as they are laterally disturbed by tidal action, then the apparent excentricity of the Nova may perhaps be accounted for in one or other of the two following ways:—

(*a*) It may be illusory. In the later pictures it has practically disappeared, and if we look back from them to the earlier ones, we can see how the boundary, especially of the inner ring, can be identified so as to make the Nova central. This identification emphasises the transverse elongation of the inner ellipse, which must be explained as above.

(*b*) The Nova may be really excentric to the spherical surfaces. In this case we must seek some physical explanation on the lines made familiar to us by recent suggestions about the "gravitational instability" of a contracting sphere. The evidence is, however, too indefinite to justify any serious consideration of such a hypothesis.

20. With the above conclusions in mind the measures given in § 6 could be revised, and those of the other photographs brought into line. This would direct attention to several interesting points: for instance, to the question whether the inner ring first expands and then contracts again, as it should if it belongs to a sphere which cuts the parabolas for the terminal dates on opposite sides of the line SB*b*. But the measurements are so delicate and uncertain that it is difficult to avoid bias, and they have been left as they were originally made with a quite erroneous hypothesis in mind.

Summary.

Consideration of the rates of expansion of the two ring-boundaries surrounding Nova Persei, as shown in Ritchey's drawings (*Ap. J.*, 15, 1902), suggests that they represent spherical surfaces concentric with the Nova. If it may be presumed that they cannot be *prolate* spheroids, we are led at once to a very definite value for the scale of the phenomena and to sensible sphericity for both surfaces. The parallax assigned for the Nova is

0″·0093 (350 light years).

It is further suggested (in line with the ideas outlined in *M.N.*, 72, p. 405) that the phenomena of the Nova were the result of a gravitational contraction of a portion of the surrounding nebula (which is disposed in nearly spherical layers, modified in one direction by tidal arms), which was very rapid in its final stages, leading to the outburst by congestion.

On the Spectrum of Nova Aquilæ (1918) *on* 1918 *June* 13 *and* 15.
By H. F. Newall, Professor of Astrophysics, Cambridge.

Amongst the photographs of the spectrum of Nova Aquilæ (1918) secured at the Solar Physics Observatory, Cambridge, two of those obtained on 1918 June 13 and 15 have been measured and studied with special reference to the numerous absorption lines which are seen in the spectral region λ4000–λ5000.

There are some points of interest which seem to justify my laying them at once before the Society.

Each photograph has a comparison spectrum of an iron spark imposed close to the stellar spectrum. The scale of dispersion is such that the distance between H_β and H_δ is 29 mm.

The Emission Bands of Hydrogen of Nova Aquilæ.

In the photograph *d*, June 13, in the series each of the emission bands of H_δ, H_γ, H_β may be described as a bright band exhibiting four maxima. Two of them, being close together and somewhat difficult to measure separately, have been bisected as a single group, which forms the brightest part of the whole band near its middle point, and, as a single group, give displacements corresponding to a mean velocity − 110 km./sec.

The other two are situated approximately symmetrically on either side of the central group; they have displacements corresponding to velocities − 1314 and + 1200 km./sec.

In the photograph *g*, June 15, of the series each of the emission bands of H_δ, H_γ, and H_β has three maxima, one central and two lateral, one on either side of the central maximum. The central

maximum here appears to be a single broad maximum. The two lateral maxima are more sharply marked than those in photograph *d*, June 13, and they have displacements of the same order of magnitude as the lateral maxima of photograph *d*, June 13.

The Displaced Absorption Lines of Hydrogen of Nova Aquilæ.

The absorption phenomena of these hydrogen lines on the violet side of the emission bands are strongly marked. The dark lines appear in a continuous spectrum which is intermediate in intensity between the emission bands and the continuous spectrum in other regions of the spectrum, where it is free from superimposed emission bands due to the enhanced lines of iron and other elements.

In photograph *d* of June 13 two marked absorption lines appear with homologous relations in the lines H_δ, H_γ, H_β, and may be described as consisting of two quite distinct parts, one very broad and highly displaced to the violet, the other narrower and less displaced to the violet.

In photograph *g* of June 13 there are three marked absorption lines with homologous relations in the lines H_δ, H_γ, H_β. Comparison of the photographs *d* and *g* suggest the idea that the broader line of June 13 might be fairly regarded as a blend of two broadish lines which on June 15 had become separated into two distinctly separated lines.

The results of measurements are summarised in Table I.

TABLE I.

Absorption Lines of Hydrogen Displacement.

		Width in A.U.	Δλ. Obs.	Δλ. Calc.	Km./sec.	Width in A.U.	Δλ. Obs.	Δλ. Calc.	Km./sec.	Width in A.U.	Δλ. Obs.	Δλ. Calc.	Km./sec.
d, June 13.	H_δ					5·5	−33·0		−2412	2·7	−22·1	−22·6	−1615
	H_γ					6·7	−35·4		−2446	4·6?	−23·8	−23·9	−1644
	H_β					8·7	−40·2		−2480	3·7	−27·6	−26·8	−1702
				Mean					−2446				−1654
g, June 15.	H_δ	1·8	−37·5	−38·3	−2741	1·2	−31·1	−31·3	−2273	2·5	−22·9	−23·1	−1673
	H_γ	?	−41·5	−40·5	−2867	1·7	−33·0	−33·1	−2280	2·7	−24·3	−24·4	−1679
	H_β	?	−45·1	−45·3	−2782		−37·6	−37·1	−2319	?	−27·9	−27·4	−1721
				Mean	−2797				−2291				−1691

Previous experience in the study of the spectra of Nova Persei (1901) and of Nova Geminorum No. 2 (1912) made me desire to test the view that the narrow absorption lines in the early spectrum of Nova Aquila are in the main ascribable to the superposition of two or even three displaced spectra, each of the type of α Cygni. The peculiar variation in the relative intensities of the narrow absorption lines in successive photographs of the early spectra of Novæ reminded me strongly of my experience in sorting out the

superimposed spectra in the binary α Aurigæ. That experience had shown that when one spectrum of many lines is periodically displaced over another similar spectrum, there arise surprising recurrent effects of both intensification and weakening or even obliteration of lines by reason of an effect which may be tersely described as a "palisade" effect, the name being suggested by the appearances seen when one looks from a distance at two palings one behind the other.

In a note* presented to the Society on the early spectrum of Nova Geminorum No. 2, it was pointed out that the lines of the Nova coincided in large measure with lines characteristic of α Cygni, if the latter spectrum were shifted to the violet with a displacement corresponding to a velocity of about 700^{km}/sec., and it was suggested that possibly a fuller coincidence would be obtained between the lines of the Nova and the displaced lines of a spectrum intermediate between α Cygni and γ Cygni.

In the study of the spectrum of Nova Aquilæ it appeared to me that the palisade effect in two or three relatively displaced spectra of α Cygni would reproduce all the absorption lines in Nova Aquilæ.

First the wave-lengths of the narrow lines in the Nova spectra for June 13 (*d*) and 15 (*g*) were determined, and the displaced absorption lines of hydrogen were first dealt with.

The values of λ'/λ were computed for each of the homologous lines in H_δ, H_γ, and H_β, where λ' is the displaced wave-length and λ the normal wave-length. These were found to be approximately constant, and thus it would appear that an interpretation of the displacements by reference to a velocity effect is justifiable, as was found by Campbell and Wright in the case of the smaller displacement observed in the spectrum of Nova Persei (*L.O.B.*, i. 49).

The following table illustrates the results for the two photographs measured :—

TABLE II.

	Photograph *d*, June 13.		Photograph *g*, June 15.		
	$\frac{\lambda''}{\lambda}$	$\frac{\lambda'}{\lambda}$	$\frac{\lambda'''}{\lambda}$	$\frac{\lambda''}{\lambda}$	$\frac{\lambda'}{\lambda}$
H_δ	·99196	·99461	·99087	·99243	·99441
H_γ	185	450	043	259	440
H_β	174	432	072	225	426
Mean	·99185	·99448	·99067	·99236	·99436

The displacements of the absorption lines, $\lambda' - \lambda$, if they are proportional to the normal wave-lengths, are thus

$$\lambda' - = (\text{mean factor} - 1).$$

The values deduced from the mean factors of Table II are entered in Table I. in the column headed "Calc."

* Newall and Stratton, *M.N.*, **73**, 380.

The Narrow Absorption Lines of Nova Aquilæ.

The absorption lines in the Nova between H_β and H_δ were measured as completely as possible, and their wave-lengths were determined.

Next the lines in a spectrum of α Cygni, photographed with the same spectrograph, were measured, only such being picked out (to the number of 41) as were within the same range of intensities as had been measured in the Nova, and their wave-lengths were determined.

It was at once seen that the principal narrow lines of the Nova spectrum obviously coincided in many respects with α Cygni lines when these latter were displaced by amounts equivalent to the displacements of the strong absorption lines of hydrogen. In fact, if a spectrum is built up synthetically of two spectra of α Cygni, displaced from the normal in a manner similar to the displacements of the duplicate absorption lines of hydrogen in the Nova of June 13, the correspondence with the Nova is fairly complete, with all its curious vagaries in emphasis on some lines and want of emphasis on others.

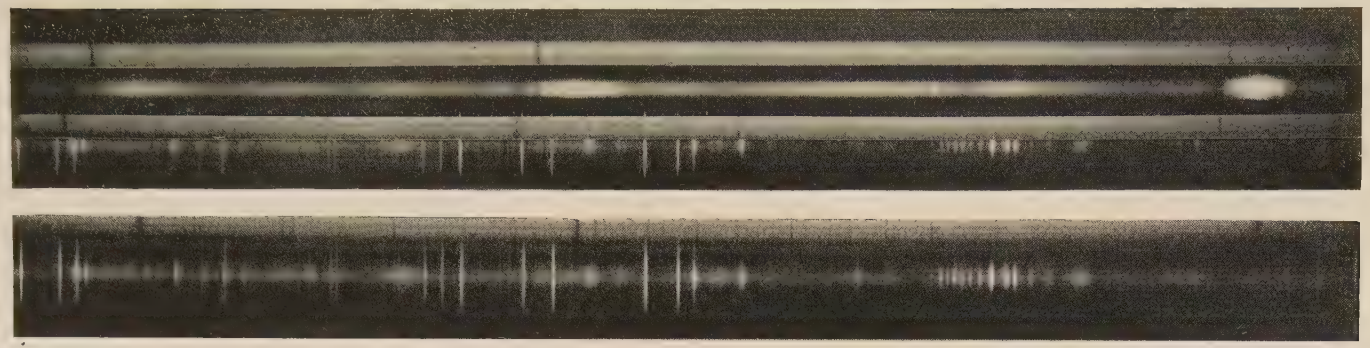
In the synthetic compounding of the spectra the displaced wave-length of each line in α Cygni was computed by multiplying the normal wave-length by factors which are the means given in Table II. Thus an α Cygni line of normal wave-length 4481·3 gives two displaced lines of wave-lengths $4481{\cdot}3 \times 0{\cdot}99185$ and $4481{\cdot}3 \times 0{\cdot}99448$; that is, of wave-lengths 4444·8 and 4456·6.

This procedure being carried out for each of the α Cygni lines, the two displaced spectra can be graphically combined to form a single synthetic spectrum to be compared with that of Nova Aquilæ on June 13. The comparison instituted shows satisfactory agreement, when account is taken of the palisade effect, which results in apparent intensification of certain lines, when, for instance, a chromium line with the smaller displacement falls on a titanium line with the larger displacement. An apparent weakening or even obliteration of lines takes place when a group of lines with the smaller displacement is out of step with the same group with the higher displacement.

In the photograph of June 13 (*d*), 50 absorption lines were measured. Out of these 38 have been clearly interpreted as α Cygni lines displaced, and 27 are connected with what may be called an α Cygni absorption with displacement corresponding to the velocity, -1654 km./sec., and 17 are connected with an α Cygni absorption with displacement corresponding to the velocity, -2446 km./sec. On examination of the $12(=50-38)$ lines which were not identified as ascribable to any of the lines actually measured by me in α Cygni, 6 have been found to be ascribable to α Cygni lines, well-known enhanced lines, but not measured by me in the α Cygni photograph.

In the case of Nova Aquilæ on June 15, when the narrow absorption lines are less accentuated, comparison is instituted

To illustrate Professor Newall's Note on Nova Aquilæ, 1918 June 13.



No. 2. 6·08-fold enlargement of the spectrum of Nova Aquilæ III., 1918 June 13.
4. 6·08-fold ,, ,, ,, Iron spark, same plate.
5. 6·08-fold ,, ,, ,, α Cygni.
6. 6·08-fold ,, ,, ,, Iron spark.

No. 1. 6·17-fold enlargement of α Cygni spectrum, its H_δ and H_β coinciding with H_δ' and H_β' of the Nova.
3. 6·26-fold ,, α Cygni spectrum, its H_δ and H_β coinciding with H_δ'' and H_β'' of the Nova.

NOTE.—Linear enlargement of α Cygni does not accurately represent the velocity-change of dispersion. But the linear enlargements used for Nos. 1 and 3 bring about displacements of H_γ of only $0^{mm}\cdot058$ and $0^{mm}\cdot116$ from the strictly appropriate positions.

between the Nova and a triple synthetic α Cygni spectrum, based on the three displacements of the hydrogen lines.

In the photograph of June 15 (*g*), 28 lines were measured; they are markedly less well defined and more difficult to measure than in the earlier plate. But 17 of them are interpreted as α Cygni lines, 8 being ascribed to the highest velocity, −2797 km./sec., 7 to the intermediate velocity, −2291 km./sec., and 9 to the lower velocity, −1691 km./sec.

In the latter photograph (*g*) an interesting case of obliteration of line absorption is to be seen between λ4426·7 and λ4445·7, a region which was noted as being a region of general reduction in the intensity of the spectrum of the Nova. In this region there are three α Cygni lines, which when superposed with the three hypothetical displacements would give a series of 9 lines so evenly spaced as to produce an obliteration.

In the same photograph (*g*) there is, however, a group of 8 nebulous lines in the region of λ4558 to λ4619, which I am at a loss to interpret.

Two interesting cases which at first sight seemed to throw weight against the view here suggested may be cited as giving confirmatory evidence in support of the view. One relates to the photograph of June 13, the other to the photograph of June 15. In the first it appeared that the very marked line corresponding with the displaced enhanced lines of iron at λ4233 was single, but on re-examination of the photograph it was found that two lines are present, one as a narrow and strongly marked line with a displacement corresponding with the less displaced lines of hydrogen, and the other as a broad diffuse line with a displacement corresponding with the more displaced and broader absorption lines of hydrogen.

In the photograph *g* of June 15 the displaced absorption lines at H_γ seemed to be fourfold, but one of them has no homologous line in H_δ and H_β. On re-examination of the computed displacements of α Cygni lines for that date, the apparent fourth component in H_γ was found to coincide with a displaced enhanced line of chromium at normal wave-length 4351, which makes its appearance between the two least displaced H_γ lines.

Conclusion.

The result of this examination leads me to believe that the hypothesis which it was desired to test is supported by the evidence extracted from the photographs which have been measured and studied. The conclusion arrived at is that narrow absorption lines in the spectrum of Nova Aquilæ on June 13 are absorption effects similar to those characteristic of α Cygni, when duplicated by two displacements corresponding with those of the absorption lines of hydrogen in the Nova.

Note.—A plate illustrating the paper is being prepared for reproduction, but is, unfortunately, not ready for the present number.

Solar Physics Observatory, Cambridge:
1918 *November* 6.

Magnitudes of Nova Aquilæ, 1918, *deduced from Photographs taken at the Royal Observatory, Greenwich.* By H. P. Hollis.

(*Communicated by the Astronomer Royal.*)

The new star in Aquila was seen at Greenwich on the nights of June 8 and 9. The first photograph from which a magnitude can be determined was taken on June 10. Since that date photographs have been taken with stars near it in position and of approximately the same magnitude for comparison on suitable nights, and the magnitudes determined from these are given in the Table which follows.

The photographs were taken with the Astrographic Equatorial, the object glass being stopped to 6 inches aperture, on Ilford Empress Plates. The stars were photographed on the same plate, each with a series of exposures differing in length, the same for each star, the exposures on the Nova being intermediate in time to those on the comparison stars. The magnitudes here given are deduced generally from exposures of 30 secs., 20 secs., 15 secs., and 10 secs. to the end of July, and of 80 secs., 40 secs., 20 secs., and 10 secs. for the later dates. The diameters of the images were measured in the duplex micrometer at first by means of the scale in the eyepiece, and from August 13 with a small micrometer mounted on the measuring machine for the purpose. From these measures of diameter, the magnitudes of the Nova have been deduced by interpolation, the magnitude of the comparison stars being taken in the first instance from the list of Standard Photographic Magnitudes of Bright Stars in *Harvard Annals*, **71**, 17, 18, and the formula, magnitude $= c - k\sqrt{\text{diam.}}$ being used. Corrections for atmospheric absorption were used in the course of the work, taken from Schwarzschild's *Aktinometrie der Stern der B.D. bis zur Grosse* 7·5, the table there given being extended to lower altitudes empirically.

ZD.	45°·5	51°·1	57°·5	63°·5
Correction	0·32	0·33	0·48	0·75 mag.

From the measured diameters of the comparison stars a determination of their relative magnitude was deduced, which depends necessarily on the value of the absorption corrections for the lower stars. A list of the comparison stars, with these magnitudes which were used as standard, is given here.

	Adopted Magnitude.	Harvard Photographic Magnitude.		Adopted Magnitude.	Harvard Photographic Magnitude.
α Lyræ	0·11	0·11	β Ophiuchi	3·97	4·11
α Aquilæ	1·07	1·07	γ Aquilæ	4·40	4·22
α Cygni	1·37	1·37	η Serpentis	4·41	4·66
α Ophiuchi	2·39	2·35	β Aquilæ	4·81	4·96
ζ Aquilæ	3·04	3·03	12 Aquilæ	5·32	5·25
θ Aquilæ	3·67	3·29	64 Serpentis	5·59	5·59*
δ Aquilæ	3·74	3·75	35 Aquilæ	6·00	5·77*

* From Revised Harvard Photometry, with Colour Index applied.

Date and G.M.T.			Magnitude of Nova.	Date and G.M.T.			Magnitude of Nova.
	d	h m			d	h m	
June	10	12 39	0·00*	Aug.	6	9 30	4·07
	11	12 0	0·44		7	10 30	3·78
	12	12 14	0·65*		10	11 18	4·14
	13	11 23	0·56		13	10 12	4·65
	15	10 18	1·47		15	9 48	4·36
	22	11 2	2·08		20	9 4	4·47
	23	11 20	2·54		21	9 20	4·57
	24	11 43	2·81		28	8 45	4·67
	25	11 16	2·77	Sept.	2	9 30	5·12
	29	10 35	3·53		6	8 38	4·50
July	1	11 14	4·03*		7	10 18	4·92
	2	11 45	3·19		10	9 36	5·26
	3	11 21	3·20		12	9 44	5·21
	6	10 36	3·25		15	8 25	5·02
	8	10 40	3·25		23	8 3	5·50
	9	10 46	3·48		25	7 40	5·59*
	10	10 50	3·58		28	8 0	5·40
	12	10 18	3·59	Oct.	7	7 13	5·36
	18	10 26	3·47		8	8 13	5·32
	21	10 13	4·43		18	7 34	5·81*
	23	10 31	4·28		22	7 4	5·55
	29	10 50	4·02	Nov.	1	7 47	5·54
	31	10 25	4·03		7	6 45	5·77

* These determinations are of small weight. The magnitude 0·00 for June 10 is merely an estimate.

Magnitudes of Nova Aquilæ. By Charles Clifford Conroy, A.M., Sc.D.

At 7.40 p.m. Standard Mean Pacific Time, 1918 June 9, I saw Nova Aquilæ near the horizon, shortly after it had risen. I sent word at once, by telegraph, to the Lick Observatory, but my discovery was, of course, too late to claim priority.

I have been fortunate in obtaining a good series of observations for magnitude, which I submit herewith. The nights, with very few exceptions, have been fine and dry this summer. My comparison stars are α Lyræ, H.R.P., mag. 0·14; α Aquilæ, 0·89; α Cygni, 1·33; λ Scorpii, 1·71; α Ophiuchi, 2·14; γ Cygni, 2·32; γ Aquilæ, 2·80; δ Aquilæ, 3·44; λ Aquilæ, 3·55; θ Serpentis, 4·10; 12 Aquilæ, 4·15; η Scuti, 5·00; 4 Aquilæ, 5·07. The observations were made with the naked eye, a binocular of 7

diameters, and a 3-inch Bardou, with which a power of 120 was employed. Moonlight was avoided as much as possible, and all observations were made near the meridian. Here I give the results as I have reduced them, up to (and including) August 15:—

Date	Civil Time. h	Mag.	Date.	Civil Time. h	Mag.	Date.	Civil Time. h	Mag.
June 9	23½	−0·75	June 26	22½	3·44	July 26	21½	4·15
10	23½	0·00	27	22½	3·80	27	21½	4·00
11	23	+0·35	July 3	21½	2·95	29	23	4·10
12	23	0·56	4	22	2·95	30	23	4·10
13	23½	1·00	5	23	2·95	Aug. 1	22	4·70
14	23	1·33	7	22½	3·30	2	21¼	4·30
15	22 45^{m}	1·60	8	21½	3·36	4	21⅝	4·70
16	23	1·75	9	22	3·44	6	21¾	3·60
17	23	2·00	10	22	3·60	7	22½	3·75
18	23	2·06	11	22	3·95	8	20⅝	3·75
19	22½	2·32	15	21¼	3·60	11	20⅝	4·15
20	23	2·50	16	21¾	3·60	12	21¼	4·10
21	22½	2·60	17	22½	3·44	13	21	4·45
23	22½	2·80	18	21½	4·00	15	21	5·00
24	22½	2·80	19	21⅝	4·00			
25	21½	2·88	25	21	4·10			

It will be noticed that between June 27 and July 3 there was a recovery of nearly one magnitude, and between August 3 and August 6 an even greater recrudescence of light. Several minor oscillations are also in evidence.

My colour notes follow:—

June 9, much bluèr than Vega; June 10, blueness still marked; June 11, not so blue as Vega; June 12, similar to Altair, steely blue of Vega wanting.

June 13, yellower than Altair; June 14, same; June 15, slight rosy tinge perceptible to naked eye.

June 16, dull yellow, with reddish cast; June 17, rosy tinge; June 18, same; June 19, rose gave place to a more fiery hue; June 20, same colour; June 21, same.

June 23, deeper hue than that of γ Aquilæ. Light not as sharp as that of neighbouring stars; June 24 and 25, same; June 26, very red. This note is repeated until July 7, when the Nova was deep yellow rather than red. On July 8 the same colour was remarked, but on July 9 the deeper hue reasserted itself, and continued until July 26, when another paling was noticed. On August 7 it was "not at all reddish," and the same remark applies to the four following observations. On August 15 it seemed to be devoid of any pronounced hue, and was leaden white.

Los Angeles, California:
1918 *August* 18.

Supplementary Note.

		h				h				h	
Aug.	24	21	4·70	Sept.	4	21	4·35	Sept.	25	19¼	5·30
	25	21¼	4·55		5	20	4·50		29	19¼	5·60
	27	21	4·30		6	21	4·50	Oct.	4	20½	5·75
	28	21	4·10		7	20½	4·50		16	20½	5·88
	30	20½	4·60		8	20	4·50		21	19½	5·75
Sept.	1	21	5·20		9	20	4·65		24	19	6·00
	2	20½	5·07		11	20	4·65		28	19	5·70
	3	20½	4·57		22	19¼	4·92	Nov.	1	19	6·00

The star was noted "not red" on August 24; slightly reddish, August 25; very slight flush, September 1; slight flush, September 2; "leaden," September 3; dull or leaden white, September 5; slight flush, September 9; "leaden," September 25; slightly reddish, October 4; no appreciable colour, November 1.

1918 *November* 4.

Observations of the Light Variation of Nova Aquilæ, 1918.

By R. Watson.

(*Communicated by Sir Frank W. Dyson, F.R.S.*)

The following observations were forwarded to Sir Frank Dyson by Dr. S. S. Hough, H.M. Astronomer at the Cape, with the request that he would communicate them to the Royal Astronomical Society if he thought that course desirable.

Dr. Hough writes:—"I am sending you herewith a copy of a series of observations of Nova Aquilæ made by Mr. R. Watson, of the Telegraph Service at Beaufort West, Cape Province, South Africa, and a member of the Cape Astronomical Association.

"Mr. Watson communicated his discovery to me by telephone at about 9 hours G.M.T. on the night of June 8. The night had been overcast here [Royal Observatory, Cape of Good Hope], but as there was a patch of clear sky visible I turned a telescope on to the position indicated by him. I caught a glimpse of the star through a temporary break in the clouds, but no useful observations could be made until the following night, June 9, when we got our first spectrogram."

[Table

Nova Aquilæ, 1918. *Light Variation.*

Date 1918.	G.M.T. h m	Deduced Magnitude.	Estimate.	Remarks.
June				
8	7 53	2·0	Brighter than α Ophiuchi	but not much. Altair not seen.
...	14 0	0·8 ?	,, ,, Altair.	
9	7 0	−0·13	$\frac{2}{3}$ Canopus to Arcturus.	
10	7 0	+0·24	= Arcturus ?	
12	8 20	0·8	Seems brighter than Altair	Cloudy.
13	...	...	...	Not much change.
14	13 45	0·85	Just brighter than Altair.	
16	12	2·3	$\frac{4}{10}$ α Ophiuchi to η Ophiuchi	Great difficulty in comparing with α Ophiuchi.
17	13 45	..	,,	Cloudy.
18	11	2·2	$\frac{2}{10}$ α to η Ophiuchi.	
19	12 0	2·1	= α Ophiuchi and = σ Sagittarii	Sky very bad.
20	10	2·1	,, ,,	
21	15 30	1·95	Possibly = ϵ Sagittarii	Cloudy.
22	12	2·1	,, = α Ophiuchi and σ Sagittarii	,,
23	16	?	Moon and clouds	? seen but not so easy as α, β, and γ Aquilæ, which were much higher.
25	11 20	2·8	? = to γ Aquilæ	Moon and clouds.
26	6 0	3·4 ?	= η Serpentis. Between π and ξ Sagittarii. Brighter than 72 Ophiuchi.	
27	9 15	3·7	$\frac{1}{2}$ way η to θ Serpentis.	
28	8 15	4·1	$\frac{1}{5}$ θ Serpentis to 6 Aquilæ.	
...	9 0	3·9	$\frac{5}{8}$ η to θ Serpentis	Moon up higher.
29	6 0	3·7	$\frac{1}{2}$ η to θ Serpentis.	
30	6 50	3·6	$\frac{3}{10}$ η to θ Serpentis.	
...	9 0	3·4	= η Serpentis or brighter.	
July				
1	6 20	3·2	$\frac{3}{5}$ β Ophiuchi to η Serpentis.	
...	8 50	3·3	$\frac{4}{5}$,, ,,	Misty.
2	6 50	3·3	,, ,, ,,	
3	7 0	3·1	$\frac{2}{5}$ β ,, ,,	
4	9 15	3·18	$\frac{1}{2}$ β Ophiuchi to η Serpentis; $\frac{1}{2}$ ζ to δ Aquilæ.	
5	7 0	3·3	$\frac{4}{5}$ β Ophiuchi to η Serpentis.	
6	8 45	3·4	= η Serpentis.	
7	10 30	...	About same	Cloudy.
8	7 0	3·4	Same.	
9	7 0	3·35	Trifle brighter or = η Serpentis	

Nova Aquilæ, 1918. *Light Variation*—continued.

Date 1918.	G.M.T.	Deduced Magnitude.	Estimate.	Remarks.
July	h. m			
10	7 0	3·4	Same.	
12	7 0	3·76	$\frac{3}{5}$ η to θ Serpentis; $\frac{2}{5}$ λ Aquilæ to θ Serpentis.	Haze and cloud.
13	7 0	3·6	$\frac{2}{5}$ η to θ Serpentis; $=\lambda$ Aquilæ.	
14	14 15	3·5	? brighter than λ Aquilæ	Misty clouds.
15	7 0	3·4	$\frac{9}{10}$ β Ophiuchi to η Serpentis $=\eta$ Serpentis.	
...	9 30			
16	10 0	3·5	$\frac{1}{5}$ η to θ Serpentis.	
17	14 0	3·5 ?	...	Seen through clouds.
18	14 0	3·7	$\frac{1}{4}$ λ Aquilæ to θ Serpentis.	
20	13 45	3·9	$\frac{3}{4}$,, ,,	
21	8 0	4·2	$\frac{1}{5}$ θ Serpentis to 4 Aquilæ	Moon strong.
22	6 0	4·0	$=\theta$ Serpentis.	
23	8 0	3·9	$\frac{3}{4}$ λ Aquilæ to θ Serpentis.	
24	5 15	3·9	,, ,,	
26	5 30	3·8	$\frac{1}{2}$,, ,,	
27	7 0	3·7	$\frac{1}{4}$,, ,,	
28	8 15	3·73	$\frac{1}{3}$,, ,,	
29	6 0	3·86	$\frac{2}{3}$,, ,,	
Aug.				
2	9 0	4·22	$\frac{1}{6}$ θ Serpentis to 4 Aquilæ.	
3	7 0	4·44	$\frac{2}{5}$,, ,,	
4	7 0	4·77	$\frac{7}{10}$,, ,,	
5	10 30	4·88	$\frac{4}{5}$,, ,,	
6	7 30	4·44	$\frac{2}{6}$,, ,,	
7	0 0	4·0	$=\theta$ Serpentis.	
8 & 9	...	4·0	,,	
10	8 0	4·11	$\frac{1}{10}$ θ Serpentis to 4 Aquilæ.	
11	9 30	3·9	$\frac{1}{4}$ λ Aquilæ to θ Serpentis	Cloudy.
12	9 0	4·11	$\frac{1}{10}$ θ Serpentis to 4 Aquilæ.	
13	9 30	4 55	$\frac{1}{2}$,, ,,	
14	6 45	4·1	$\frac{1}{11}$,, ,,	
15	11 15	4·88	$\frac{4}{5}$,, ,,	
16	7 0	4·5	$\frac{5}{11}$,, ,,	Moon.
17	10 30	4·2	$\frac{2}{11}$,, ,,	
18	9 0	...	? $=\theta$ Serpentis	Moon bright.
19	7 15	3·95	$\frac{1}{8}$ λ Aquilæ to θ Serpentis	,, ,,
21	6 45	4·4	$\frac{4}{11}$ θ Serpentis to 4 Aquilæ.	

Nova Aquilæ, 1918. *Light Variation*—continued.

Date. 1918.	G.M.T.	Deduced Magnitude.	Estimate.	Remarks.
Aug.	h m			
22	5 15	4·4	$\frac{4}{11}$ θ Serpentis to 4 Aquilæ.	
...	6 0	4·2	$\frac{2}{11}$,, ,,	
...	9 15	4·0	= θ Serpentis	Moon and mist.
23	5 15	4·4	$\frac{4}{11}$ θ Serpentis to 4 Aquilæ.	
...	9 15	4·6	$\frac{6}{11}$,, ,,	
24	5 30	4·55	$\frac{1}{2}$,, ,,	
...	7 0	4·5	$\frac{5}{11}$,, ,,	
...	8 0	4·6	$\frac{6}{11}$,, ,,	Moon risen.
25	8 15	4·77	$\frac{7}{10}$,, ,,	
26	6 35	4·4	$\frac{4}{11}$,, ,,	Misty.
...	8 45	4·7	$\frac{7}{11}$,, ,,	
27	5 25	4·33	$\frac{8}{10}$ θ Serpentis to 4 Aquilæ.	
...	8 35	4·55	$\frac{1}{2}$,, ,,	
28	5 35	4·0	= to θ Serpentis.	
...	8 20	4·55	$\frac{1}{2}$ θ Serpentis to 4 Aquilæ.	
29	5 40	4·7	$\frac{7}{11}$,, ,,	
...	8 45	4·8	$\frac{8}{11}$,, ,,	
30	5 30	4·7	$\frac{7}{10}$,, ,,	
...	8 30	4·9	$\frac{9}{11}$,, ,,	
31	8 25	4·96	$\frac{7}{8}$,, ,,	
Sept.				
1	...	4·96	$\frac{7}{8}$,, ,,	
2	8 20	5·0	$\frac{10}{11}$,, ,, ?	
3	9 30	4·8	$\frac{1}{2}$ 6 Aquilæ to 4 Aquilæ.	
4	6 0	4·8	$\frac{1}{2}$,, ,,	
5	6 30	4·74	$\frac{4}{10}$,, ,,	
6	5 30	4·58	$\frac{5}{11}$ θ Serpentis to 4 Aquilæ (4·5). $\frac{1}{4}$ 6 Aquilæ to 4 Aquilæ (4·65).	
7	7 0	4·5	$\frac{5}{11}$ θ Serpentis to 4 Aquilæ (4·5). =or brighter than 6 Aquilæ (4·5).	
8	6 45	4·65	$\frac{8}{11}$ θ Serpentis to 4 Aquilæ (4·8). =or brighter than 6 Aquilæ (4·5).	
12	7 50	4·6	$\frac{7}{11}$ θ Serpentis to 4 Aquilæ (4·7). =or brighter than 6 Aquilæ (4·5).	

Notes.—Magnitudes taken from Backhouse Catalogue. Observations brighter than magnitude 4·0 are naked eye ; below that, finder was used.

The Position of Nova Aquilæ. By R. J. Pocock, B.A., B.Sc.

The weather has been very cloudy at Hyderabad since the appearance of Nova Aquilæ, but two plates of the region have been secured with the 8-inch Cooke telescope, on 1918 June 15 and 1918 July 10. The former of these is not very good, and has not been used in the present investigation; the second plate (No. 1237) was exposed by Mr. Bappu, and has two exposures of 12 and 20 minutes respectively. Owing to an error in setting, the plate was not centred on the Nova, the centre being approximately at R.A. $18^h\ 46^m$, N. Decl. 0° 50′. Both exposures have been measured by myself, and the plate constants determined by comparison of the measured co-ordinates of 23 stars with the positions given in the catalogues of the *Astronomische Gesellschaft* (Albany and Nicolajew) brought up from 1875·0 to 1900·0.

With the above plate centre the constants for the two exposures become—

Exposure.	A.	B.	C.	D.	E.	F.	Hour Angle.
m							h m
12	−·01740	−·00311	+1·4485	+·00322	−·01754	−·1277	0 6 E.
20	−·01744	−·00306	+1·4396	+·00322	−·01754	−·0073	0 11 W.

The réseau used for this plate is a good one by Gautier recently lent to us by the Kodaikánal Observatory.

The mean residuals from the two solutions are shown for each of the reference stars below (in the sense Hyd.–A.G.C.) in units of 0″·03; it should be remembered, however, that the measures are only made to 0″·3, but the means of the measures in the two positions of the plate were taken to 0·0001 of a réseau interval equivalent to 0″·03, and all subsequent work done to this order. The four stars marked with an asterisk were not used in forming the solution, since they appeared to give large residuals; none of the 23 reference stars used gives a residual as large as 1″·5.

Residuals of Reference Stars from Hyderabad, Plate No. 1237.

Star.	Δx.	Δy.	Star.	Δx.	Δy.
	Unit 0″·03.			Unit 0″·03.	
N 4678	−11	+26	N 4698	− 9	+ 30
A 6377	+ 4	−18	N 4699*	−150	− 18
A 6379	+22	+42	N 4701	+ 40	+ 4
N 4681	−21	+ 6	A 6392	− 7	+ 1
N 4682	+12	+41	N 4707	− 16	− 18
N 4684*	−46	−82	A 6401	− 36	− 6
N 4685	+35	+14	N 4709*	− 50	− 84
N 4686	−32	−38	A 6406	+ 34	+ 40
N 4687	+ 4	+30	N 4712	− 36	+ 19
N 4690	−26	10	A 6413*	+ 60	−106
N 4691	−36	−22	A 6414	+ 46	− 48
N 4692	+30	−18	N 4720	− 14	− 6
N 4693	+ 6	−26	N 4721	− 4	− 15
N 4694	+12	32			

In every case the residuals from the two solutions agree within the errors of measurement, the mean difference without regard to sign being 0″·21 in x and 0″·27 in y; the inferior result in y is due to two stars (N 4678 and N 4720), for which the differences are 0″·69 and 0″·81 respectively (in opposite senses); the largest difference in x is 0″·48.

Using the plate constants given above, the position of the Nova, deduced from several sets of measures of the two exposures made on three different days, is—

Exposure.	R.A. 1900·0.			Decl. 1900·0.		
m	h	m	s	°	′	″
12	18	43	48·314	+0	28	20·97
20	18	43	48·312	+0	28	20·91
Mean	18	43	48·313	+0	28	20·94

The plate does not show a great many faint stars, but besides N 4685 there are two stars of about the tenth magnitude not far from the Nova; their positions for 1900·0 are—

h	m	s	°	′	″	
18	42	52·608	+0	28	41·08	1900·0
18	43	34·838	+0	32	8·25	

The following observations of magnitude and colour were made while the Nova was bright. The observations were made at Kodaikánal; the times given are Indian standard, $5^h\ 30^m$ east of G.M.T:—

June 11. Nova slightly brighter than Altair, 0·6 mag., white.
June 12. Nova = Altair, white.
June 13. Altair > Nova > α Cygni, 1·1 mag., yellow.
June 15. Nova distinctly less than Altair, approx. 1·3 mag.
June 18. $21^h\ 30^m$. α Scorpii > Nova > β Scorpii, Nova = η Ursæ Maj., 1·8 mag., red.
June 19. $21^h\ 30^m$. ζ Ursæ Maj. > Nova > β Scorpii, 2·6 mag., red.
June 20. $21^h\ 30^m$. Nova appears same as last night, but much cloud about, hindering observations.
June 21. $22^h\ 0^m$. Cloudy. Nova 0·3 or 0·4 brighter than ζ Aquilæ.
June 23. Nova slightly brighter than ζ Aquilæ, much fainter than γ Ursæ Maj., red.

Nizamiah Observatory, Hyderabad:
1918 *July* 24.

The Relation between Mean Parallax and Magnitude.

By R. J. Pocock, B.A., B.Sc.

1. The first-fruits of the new spectroscopic method of determining parallax have recently been published in the form of a list of luminosities and parallaxes of five hundred stars.* It appears that these results are being used at Mount Wilson to determine a new formula for the mean parallax of stars of determined magnitude and proper motion;† there is no intention of trespassing on this ground in the present paper, but it occurred to the present writer that it might be of interest to examine the relation between mean parallax and apparent magnitude on the basis of this material, the first intention being to see how far the very simple formula ‡ $m \cdot \varpi = c^2$ would fit these results.

2. Before proceeding to discuss the results it is proper to consider how far it is legitimate to use the material for this purpose. In the first place, it may be urged that both the number of stars and range of magnitude are small, and certainly in this respect the material compares very unfavourably with the great mass of observations recently discussed with the same end in view by Dyson and Thackeray.§ Nevertheless the present material is by no means insignificant, and the fact that it is possible to separate the different spectral classes, and in particular to differentiate between the giant and dwarf stars, seems to make it worthy of investigation. A more serious criticism may be that is is not proper to make use of the measured parallaxes at all for the present purpose. The objection to the use of measured parallaxes is that they are a very prejudicial selection, not representative of the stars as a whole, including much too large a proportion of large parallaxes. It appears, however, that in the present instance this criticism has not its usual force; for although the list of five hundred stars was chosen so as to include as many measured parallaxes as possible, almost one-third of the number (140 out of 500) have no measured parallax, and the 360 for which parallaxes are available include a considerable number of stars having very small parallaxes, measured by van Maanen. In this connection the remarks on pp. 335, 336 of the paper cited may be referred to, from which we may deduce that the observing list was not unduly overbalanced by stars of large parallax. If these two objections can be overcome, the comparison of a formula derived from measured (spectroscopic) parallaxes with that derived from parallactic motions is of some interest.

3. As soon as the mean parallaxes were determined for the different magnitudes it was seen that the formula given above, viz. $m \cdot \varpi = c^2$, quite failed to represent the observations even over

* *Astrophysical Journal*, **46**, 313.

† See *Astrophysical Journal*, **45**, 298; **47**, 8.

‡ A. S. Eddington, *Stellar Movements and the Structure of the Universe*, p. 207.

§ *Monthly Notices, R.A.S.*, **77**, 7.

the short range of magnitude involved; as examples the values of the product $m \cdot \varpi$ are given below for the F stars and the giants ($M < 4{\cdot}0$)* of type K.

Magnitude (m).		2·5.	3·5.	4·5.	5·5.	6·5.	7·5.	8·5.
$m\varpi$.	F stars	...	·262	·283	·280	·260	·218	·178
	K giants	·108	·129	·104	·082	·071	·037	...

The product $m \cdot \varpi$ falls off rapidly with increase in the value of m. A much better agreement could be obtained by increasing the exponent of m, equivalent to writing

$$\log \varpi_m = a + b \log m \qquad b < -1,$$

but this formula has no advantage over Kapteyn's form, viz.

$$\log \varpi_m = a + b \cdot m.$$

4. It seemed therefore of interest to compare the results with Kapteyn's formula. Table I. gives the values of $\log(1000 \cdot \varpi)$ for the different groups of stars; the division into giants and dwarfs is made at $M = 4{\cdot}0$ for the K stars and $M = 2{\cdot}4$ for the G stars: the M dwarfs were too few for consideration. The suffixes denote the numbers of stars in each group; values in brackets were not used in deducing the formulæ.

TABLE I.

Values of log (1000 . ϖ) *from List of Spectroscopic Parallaxes.*

Type. \ Mag.	3·5.	4·5.	5·5.	6·5.	7·5.	8·5.	9·5.
F, all stars	$1{\cdot}875_{20}$	$1{\cdot}799_{23}$	$1{\cdot}708_{33}$	$1{\cdot}602_{27}$	$1{\cdot}462_{20}$	$1{\cdot}322_{9}$	$(1{\cdot}146_{2})$
G dwarfs	...	$2{\cdot}013_{9}$	$1{\cdot}778_{10}$	$1{\cdot}732_{25}$	$1{\cdot}690_{17}$	$1{\cdot}602_{18}$	$(1{\cdot}362_{2})$
G giants	$1{\cdot}477_{23}$	$1{\cdot}342_{14}$	$1{\cdot}000_{12}$	$0{\cdot}903_{10}$	...	...	...
K dwarfs	...	...	$2{\cdot}079_{6}$	$2{\cdot}021_{10}$	$1{\cdot}886_{24}$	$1{\cdot}778_{29}$	$(1{\cdot}919_{14})$
K giants	$1{\cdot}568_{11}$	$1{\cdot}362_{12}$	$1{\cdot}176_{22}$	$1{\cdot}041_{21}$	$(0{\cdot}699_{3})$	...	...
M giants	$1{\cdot}491_{7}$	$1{\cdot}447_{6}$	$1{\cdot}204_{10}$	$1{\cdot}114_{7}$	...	...	...

It will be noted that there are practically no giant stars less than the seventh magnitude; giant stars of apparent magnitude less than 7·0 will nearly all have parallaxes which only become significant in the fourth decimal place, whereas the Mount Wilson observers confine themselves to three places of decimals; there seems no justification for omitting the faint K dwarfs, except that their inclusion makes it impossible to determine a satisfactory formula.

5. The formulæ obtained for the six groups of stars are given below, and in Table II. the differences between the computed and observed parallaxes.

* M represents absolute magnitude; except where it is used to designate a spectral type.

Parallax Formulæ.

F, all stars	$\log \varpi_m = -0\cdot706 - 0\cdot111\, m$	(1)
G dwarfs	$= -0\cdot568 - 0\cdot103\, m$	(2)
G giants	$= -0\cdot865 - 0\cdot191\, m$	(3)
K dwarfs	$= -0\cdot359 - 0\cdot100\, m$	(4)
K giants	$= -0\cdot762 - 0\cdot192\, m$	(5)
M giants	$= -1\cdot056 - 0\cdot126\, m$	(6)

TABLE II.

Computed – Observed Parallaxes.

Unit 0″·001.

Mag. / Type.	3·5.	4·5.	5·5.	6·5.	7·5.	8·5.	9·5.
F, all stars	+6	− 1	− 3	−3	0	+1	(+ 3)
G dwarfs	...	−10	+13	+4	−3	−4	(+ 5)
G giants	−1	− 3	+ 2	0	...	...	...
K dwarfs	...	...	+ 3	−7	+1	+2	(−34)
K giants	0	+ 1	0	−1	+1	...	...
M giants	+1	− 4	+ 2	0	...	...	...

It is proper to remark that in each case we have from four to six observations to satisfy with two disposable constants; it would therefore be surprising if there was not a fair measure of agreement between the computed and observed parallaxes. Nevertheless, when we consider the small number of stars involved in some of the magnitude intervals, the agreement must be considered as distinctly good. The agreement is poor for the G type dwarfs; for all the other groups it seems quite good.

6. When we come to consider the formulæ (1) to (6), one or two interesting facts emerge. It will be noted that formulæ (3) and (5), which refer to the giants, agree in giving a large coefficient of m. Kapteyn's factor* is 0·108, and Dyson and Thackeray† found the value 0·120. The large value obtained in formulæ (3) and (5) is due entirely to the bright stars as shown in Table III., in which the observed mean parallaxes are compared with the values computed from Kapteyn's formula, viz. $\log \varpi_m = -1\cdot275 - 0\cdot108m$.

TABLE III.

Observed Parallaxes compared with Kapteyn's Formula.

Mag.	3·5.	4·5.	5·5.	6·5.	7·5.
G giants	·030	·022	·010	·008	...
K giants	·037	·023	·015	·011	·005
Kapteyn's formula	·022	·017	·014	·011	·008

* When reduced to the modern value of the solar motion.

† *Monthly Notices, R.A.S.*, 77, 13.

It seems probable therefore that among the giants in the list of apparent magnitude brighter than 5·0 there are too many excessive parallaxes, and the mean values are thereby vitiated for the present purpose; on the other hand, it may be that the large parallaxes are actually in excess of any formula deduced from fainter and more distant stars, corresponding to the excess of large proper motions usually found in statistical investigations; in this case we must reject the brighter stars, and in that case formulæ (3) and (5) would have to be determined from too short a range to be of any value. The M giants do not seem to be affected in this way to any appreciable extent, in which connection the remarks by Adams and Joy on the selection of M type stars may be referred to.*

Considering the scantiness of the material and the short range of magnitude, the agreement of the remaining four formulæ is excellent; the mean value of the coefficient of m from these four formulæ is $-0\cdot110$, in close agreement with Kapteyn's value.

7. The much smaller value of the constant term, particularly in formulæ (1), (2), and (4), is of course due to the separation of dwarfs and giants. In Dyson and Thackeray's investigation, for example, both dwarfs and giants were included, with naturally a preponderance of the latter, at any rate among the brighter magnitudes. Now Adams and Joy find the following differences between the magnitudes of maximum frequency for giants and dwarfs in the various classes :— *

Type.	F.	G.	K.	M.
Difference in M	3·0	4·7	5·7	9·2
Difference in log ϖ	0·6	0·9	1·1	1·8

If we assume that two-thirds of these differences have to be added to the above formulæ to make them comparable with Dyson and Thackeray's result (the figure two-thirds being a rough allowance for the preponderance of giant stars in their material), formulæ (2) and (4) become

$$\log \varpi_m = -1\cdot168 - 0\cdot103m$$

and

$$\log \varpi_m = -1\cdot159 - 0\cdot100m,$$

which must be approximately comparable with Dyson and Thackeray's formula $\log \varpi_m = -1\cdot224 - 0\cdot120m$.

The constants are now of the same order, and this may be taken as evidence that the number of large parallaxes in these two groups is not excessive.

8. The present investigation is purely tentative, but it suggests that the spectroscopic parallaxes will soon furnish material suitable for a direct determination of the mean parallax of stars of different magnitudes.

* *Astrophysical Journal*, **46**, 335.

So far as the present results go they appear to show that—

(1) The formula $m \cdot \varpi = c^2$ will not suit the observations at all.

(2) The observations are well suited by a formula of Kapteyn's form, but, either from real or observational causes, the mean parallax of the brighter stars is larger than that given by the formula.

(3) For the dwarf stars and the fainter giants, *i.e.* all stars fainter than about the fifth magnitude, the agreement with Kapteyn's formula is excellent, and in particular Kapteyn's value of the rate of increase of distance with decreasing apparent brightness is confirmed.

Nizamiah Observatory, Hyderabad:
1918 *July* 31.

Stars with Large Proper Motions in the Hyderabad Astrographic Zones. First List. By R. J. Pocock, B.A., B.Sc.

In the course of the Hyderabad astrographic work, the places of the stars in the Washington and Algiers A.G. Catalogues are compared with the photographic places in order to deduce the plate constants. These constants are then applied to the photographic places and the resulting co-ordinates compared with the meridian positions. The differences between the two are as a rule small, and probably due to accidental errors or small systematic errors, such as magnitude equation. When a large difference is found there are three possible explanations—(*a*) error in the Hyderabad position, (*b*) error in the meridian catalogue, (*c*) proper motion.

As regards (*a*), where two or more plates both give large and concordant residuals there is little possibility of error, whilst where only one plate is available, by carefully checking the measures and reductions, it is possible to ensure that no error exists. As regards (*b*), the present list depends only on comparisons Hyderabad-Washington, and all our experience goes to show that the accuracy of the Washington Catalogue is very high, nearly all the residuals are small, and of the larger ones a large proportion are known proper-motion stars, while in the comparison of nearly 400 plates with Washington only one certain case of error has so far been found.*

In compiling the following list, therefore, all stars which showed a residual of 10 units (equivalent to $3''{\cdot}0$) in either co-ordinate have been included, all due care being taken to eliminate any possibility of mistake in the Hyderabad measures or computations.

Stars, however, occurring in the well-known lists of Boss,† Porter,‡ and Van Maanen§ have been omitted.

The list of proper motions given at the beginning of the

* *A.N.*, 4833.
† *Preliminary General Catalogue.*
‡ *Cincinnati Publications*, 12.
§ *Astrophysical Journal*, 41, 187, 1915.

Washington Catalogue is very deficient, omitting many well-known proper-motion stars, and in other cases giving very incorrect values of the motion.

The average interval between Washington and Hyderabad is about twenty years, and the following list contains all stars in the Washington Catalogue which show a motion in this interval of 3″·0 in either co-ordinate (excepting those given by Boss, etc.) south of 15° 55′ S. Dec. (1900·0).

The following table needs little explanation. The magnitudes in column 2 are those given in Washington, and are therefore on the B.D. scale. In columns 3 and 4 the differences Hyderabad-Washington are given in units of 0″·3; the sign + in column 4 denotes that the south declination is increasing.

List of Large Proper-Motion Stars in the Washington A.G.C. south of 15° 55′ (*excepting those given in Boss's, Porter's, and Van Maanen's Lists*).

Washington. No.	Mag.	Δx.	Δy.	Centl. P.M. in Arc.	No. of Plates.	Interval in Years.
				″		
126	9·1	−28	+16	44·79	2	21·6
161	8·9	+10	− 6	16·20	1	21·6
228	8·0	− 9	+13	21·96	2	21·6
275	8·8	+20	− 1	27·81	1	21·6
304	9·2	+13	+ 6	20·97	2	20·5
324	9·1	+15	+29	45·35	1	21·6
435	8·4	+18	+ 9	28·20	1	21·4
448	9·1	+10	− 7	17·04	2	21·5
471	8·4	+10	+22	33·90	1	21·4
490	8·6	+16	+ 9	25·98	1	21·2
523	9·2	+ 1	−13	18·30	1	21·4
529	9·0	+ 9	−12	21·03	1	21·4
599	9·2	+10	+ 6	16·20	1	21·6
732	9·2	+16	+ 9	28·11	1	19·6
901	7·7	+14	+ 2	20·20	2	21·0
1002	9·0	− 2	+10	15·30	2	20·0
1105	8·6	+ 3	−17	26·02	1	19·9
1117	9·0	− 6	+17	26·13	3	20·7
1344	9·6	+ 1	+10	15·39	2	19·6
1504	7·9	+18	+12	33·00	1	19·7
1545	8·6	+11	− 9	21·75	1	19·6
1556	9·1	+ 4	+15	25·87	1	18·0
1639	8·0	+ 4	+10	16·58	1	19·5
2264	9·2	+ 1	+12	19·86	1	18·2
2513	8·6	+12	+10	24·03	2	19·5
2624	9·5	− 7	+11	18·81	1	20·8

List of Large Proper-Motion Stars—continued.

Washington. No.	Mag.	Δx.	Δy.	Centl. P.M. in Arc.	No. of Plates.	Interval in Years.
				″		
2656	9·0	− 12	+ 7	20·94	1	19·9
2776	7·6	− 4	− 13	20·10	1	20·3
2785	8·2	− 5	− 10	16·95	2	19·9
3459	8·8	− 10	− 4	15·84	2	20·4
3498	9·3	28	+ 26	55·75	1	20·2
4009	9·0	− 16	− 2	24·93	2	19·4
4107	9·2	− 11	− 1	16·26	1	20·4
4149	9·0	− 35	− 10	53·53	2	20·4
4286	8·4	− 18	+ 4	28·23	1	19·6
4555	9·0	+ 11	+ 5	17·85	1	20·3
4558	9·4	− 10	− 5	16·53	1	20·3
4622	8·3	− 12	− 2	17·07	2	21·4
4713	8·9	− 14	+ 4	17·40	1	21·4
4723	9·2	− 10	+ 2	14·72	2	20·8
4757	8·7	− 10	− 1	14·58	3	20·7
4792	9·0	+ 15	+ 12	27·84	2	20·7
4891	9·2	− 16	− 4	23·67	2	20·9
4943	8·6	− 11	− 4	17·39	1	20·2
4974	8·5	+ 3	− 10	17·11	2	18·3
5021	9·1	− 13	+ 6	23·22	1	18·5
5022	8·2	− 13	+ 5	24·72	1	16·9
5175	9·4	− 8	+ 16	25·20	3	21·3
5224	9·2	− 1	+ 11	15·57	1	21·3
5251	8·8	− 10	0	14·10	2	21·3
5253	9·2	− 10	+ 3	14·70	2	21·3
5273	9·0	− 19	− 2	26·91	2	21·3
5362	9·0	− 12	− 4	17·33	2	21·9
5366	9·2	− 1	+ 22	31·02	2	21·3
5562	9·6	− 12	− 1	16·96	1	21·3
5703	8·8	− 4	+ 21	30·12	2	21·3
5738	9·0	+ 14	− 5	20·94	2	21·3
5831	8·8	− 17	+ 14	31·02	2	21·3
6026	8·2	− 5	+ 18	26·31	1	21·3
6060	8·8	− 5	+ 13	19·62	2	21·3
6093	8·8	− 15	− 4	21·87	1	21·3
7081	7·9	0	− 11	16·35	1	20·2
7085	8·2	+ 10	− 3	14·85	1	21·1
7129	9·0	+ 36	+ 44	83·60	1	20·4

List of Large Proper-Motion Stars—continued.

Washington. No.	Mag.	Δx.	Δy.	Centl. P.M. in Arc.	No. of Plates.	Interval. in Years.
				"		
7346	8·4	+ 7	+15	24·34	1	20·4
7552	8·8	+14	+ 3	20·55	2	20·9
7565	8·7	+11	+25	37·93	1	21·6
7578	9·4	− 2	+10	13·92	2	22·0
7615	9·0	+ 5	+16	22·95	2	21·9
8008	9·0	+ 2	+10	14·31	1	21·4
8044	8·0	+11	+10	20·94	1	21·3
8154	8·2	+15	− 8	23·19	2	22·0
8212	8·6	+10	0	13·63	2	22·0
8217	9·0	+13	0	18·06	1	21·6
8297	9·4	−14	+ 3	19·89	1	21·6
8336	9·2	+12	+ 6	18·72	1	21·5
8463	8·0	+25	+ 4	35·17	1	21·6
8528	8·2	+ 2	+12	16·90	2	21·6
8564	9·0	+12	+ 1	16·65	1	21·7
8684	9·4	−11	+ 3	15·54	2	22·0
8688	8·1	+20	+16	34·92	2	22·0
8698	8·0	+10	−11	20·28	2	22·0
8708	9·0	+24	− 7	34·08	2	22·0
8808	8·7	+82	0	113·89	2	21·6
8813	7·9	+12	− 2	16·90	2	21·6

No. 126. The large p.m. is confirmed by comparison with Weiss-Argelander and Bordeaux Catalogues. It is also included in the list of p.m. stars in Washington.

No. 324. The large p.m. is confirmed by a comparison with Schönfeld.

No. 3498. This is 2631 in Bordeaux. A note in App. I. of that catalogue states that Weiss-Arg. requires corrections of -1^s and $-20''$ in R.A. and Dec. respectively. Since the difference in epoch Bord. W.-A. is 43·15 years, the difference in R.A. ($1^s·18$) might be accounted for by a p.m. of $-^s·027$, agreeing closely with the p.m. in R.A. from Hyd.-Wash. In the list of proper motions at the beginning of the Washington Catalogue a p.m. $-0^s·026$ is given in R.A., with the remark that Weiss-Arg. requires a correction of $-20''$ in δ. A comparison with Schönfeld also indicates a p.m. in R.A. The epochs of the Bordeaux and Washington observations of this star are almost identical.

No. 4149. This p.m. requires further confirmation, since it is not confirmed by Schönfeld.

No. 7129. This p.m. is confirmed by a comparison with

Schönfeld, but Schönfeld's declination appears to be about 20″ too small.

No. 8808. This large p.m. in R.A. is also confirmed by a comparison with Schönfeld.

Washington 658 is No. 65 in Van Maanen's list. The p.m. was originally given by Porter in *A.N.*, 4406. The observations there given are rather discordant; the Hyd. – Wash. comparison leads to a considerably smaller though still large p.m., viz. $\mu_\alpha = +0^s\cdot024$.

Washington 5795 has a p.m. $\mu_\alpha = +0^s\cdot006$ according to Washington; the differences Hyd. – Wash. show no p.m., but a comparison with other catalogues shows that the p.m. is probably correct, but the Washington R.A. is too great. We have the following positions of this star:—

Catalogue.	Epoch.	R.A. 1900·0.	Dec. 1900·0.
		h m s	° ′ ″
Weiss-Arg. . .	1850·2	15 46 31·97	18 8 10·4
Bordeaux . . .	1883·0	15 46 32·17	18 8 9·5
A.G. Algiers . .	1890·4	15 46 32·17	18 8 11·1
A.G. Wash. . .	1895·0	15 46 32·41	18 8 10·9
Hyd. (2 plates) . .	1916·8	15 46 32·43	18 8 10·0

A motion $\mu_\alpha = +0^s\cdot006$, $\mu_\delta = 0''\cdot0$ would suit all these positions except that from Washington.

Washington 6520 is No. 400 in Van Maanen's list of large p.m.'s where a proper motion $\mu_\alpha = +0''\cdot55$ is given on the authority of Washington. It occurs on only one Hyderabad plate which gives differences $\Delta_\alpha = -0^s\cdot08$, $\Delta_\delta = -0''\cdot6$, Δ. Ep. = 18·6 years; the star therefore probably has no large p.m. A note to the same effect as regards W. 5002 (Van Maanen 267) has already been published.*

In the case of the following stars the p.m.'s appear to be much smaller than the values given in Washington, viz. Wash. 111, 4734, 4830, 5288, 6429, 8260.

In the case of Wash. 5052, the Hyd. results (3 plates) indicate a small p.m. in R.A. of the opposite sign to that given in Washington. For 6035 two Hyd. plates indicate the opposite sign for μ_α and a much smaller value for μ_δ than is given in Washington.

Nizamiah Observatory, Hyderabad:
1918 *August* 22.

* *A.N.*, 4814.

The Relative Numbers and Areas of Sun-spots East and West of the Central Meridian during the Years 1902–1917. By R. J. Pocock, B.A., B.Sc.

1. In a well-known paper * Mrs. Maunder has discussed the relative numbers and areas of sun-spots east and west of the sun's central meridian during the years 1889–1901 inclusive, obtaining the result that there was a distinct eastern excess of both numbers and areas during this period.

Mr. Evershed suggested to me the desirability of continuing this investigation now that another complete sun-spot cycle has elapsed, particularly with a view to comparing the results with similar results for prominences obtained at Kodaikánal.

2. Mrs. Maunder divided the sun's visible hemisphere into fourteen lunes, of which each individual eastern lune showed an excess over the corresponding western lune. I have not thought it necessary to analyse the numbers so closely, and for the purposes of this paper have divided the solar disc into four quadrants only. The numbers of spots for these four quadrants are shown in columns 2 to 5 of Table I. The numbers for the years 1902–1913 have been taken from the sun-spot ledgers in the Greenwich volumes. For the remaining four years the numbers of spots shown on the Kodaikánal drawings have been counted. The remaining columns of the table explain themselves, except perhaps the last. This column gives the values of 100(NE + SE) / (total for year), the percentages so calculated being comparable with those given for prominences in *Kodaikánal Bulletin*, xxviii., and subsequent Bulletins. These percentages are not directly comparable with those given by Mrs. Maunder.†

TABLE I.

Analysis of Numbers of Sun-spots for the Years 1902–1917.

Year.	NE.	NW.	SE.	SW.	NE+ SE.	NW+ SW.	Total.	E–W.	Per-centage.
1902	46	52	23	23	69	75	144	−6	47·92
1903	173	167	253	200	426	367	793	+59	53·72
1904	438	391	292	274	730	665	1395	+65	52·33
1905	576	569	416	392	992	961	1953	+31	50·79
1906	610	614	306	308	916	922	1838	−6	49·84
1907	467	439	520	549	987	988	1975	−1	50·00
1908	402	315	498	494	900	809	1709	+91	52·66
1909	305	301	435	441	740	742	1482	−2	49·93
1910	126	96	316	251	442	347	789	+95	56·02
1911	40	32	120	87	160	119	279	+41	57·35
1912	2	11	76	66	78	77	155	+1	50·32
1913	26	22	7	12	33	34	67	−1	49·25
1914	75	86	69	59	144	145	289	−1	49·83
1915	312	283	229	234	541	517	1058	+24	51·13
1916	414	366	275	270	689	636	1325	+53	52·00
1917	586	591	495	490	1081	1081	2162	0	50·00
1902–1913	3211	3009	3262	3097	6473	6106	12,579	+367	51·46
1902–1917	4598	4335	4330	4150	8928	8485	17,413	+443	51·27

* *M.N.*, 67, 451 (1907 May). † *Loc. cit.*, p. 461.

The features of Table I. which seem to be of interest are the following:—

3. First, there is on the whole a small eastern excess, the number of spots observed east of the meridian being 51·46 per cent. of the whole number, or 51·27 per cent. if the last four years are included. This percentage is distinctly less than that found by Mrs. Maunder. If her percentage for the numbers of spots observed during the previous cycle is exhibited in the same form, the eastern percentage of spots becomes 52·24.

That this eastern percentage in Table I. is real seems evident from an inspection of the numbers for the individual years, from which it appears that no single year gives an appreciable western percentage (the year 1902 depends on too few spots to have any weight), whereas eight years give a very considerable eastern percentage. No one looking at the numbers in the last column but one could impute them entirely to chance.

4. Secondly, we note that the eastern excess only seems to manifest itself when the number of spots is rising or falling rapidly; when the solar activity is at a maximum or minimum there is no eastern excess. The year 1908 is apparently an exception, but reasons will be given later for assuming that this year is affected by some unascertained accidental cause.

This fact that the eastern excess is not constant, but disappears altogether at maximum and minimum, seems to negative completely the possibility of the phenomenon being caused by instrumental or observational causes. Unfortunately Mrs. Maunder has not given the numbers and areas for the individual years in her paper; if they can be made available, and they confirm the above statement, there will be very strong grounds for assuming a real physical cause for this eastern excess. Sixteen years is so short a period that it seems better to wait for further evidence before offering any suggestions as to the possible nature of this cause.

5. Thirdly, it is to be noted that the northern and southern spots behave differently as regards this eastern excess. The numbers for the individual quadrants are small, and some allowance must be made for accidental error. This difference is most notable during the last four years, when the eastern excess is entirely due to the northern spots. On the other hand, if we go back eleven years, during the years 1903-1906 the eastern excess is much more marked for the southern spots.

The eastern excess for 1908 is entirely due to the northern spots, although of the other years in the period 1905–1909 inclusive (with the exception of a much smaller eastern excess in 1907) the northern spots show practical equality. It will appear presently that this eastern excess in 1908 is not borne out by the areas, which behave just like the areas of other years showing no eastern excess of numbers, and it seems reasonable therefore to assume some cause resulting in the recording of an unusually large number of small spots in the sun's north-east quadrant during 1908.

6. It is interesting to compare the results of Table I. with the

corresponding areas, both because the areas are a more reliable index of solar activity and because any difference between numbers and areas will mean a difference in behaviour between large and small spots. I am indebted to Mr. Balasundaram Ayyar, Fourth Assistant at the Solar Physics Observatory, Kodaikánal, for assisting me by counting the areas for the three years 1906–07–08 by Mr. Evershed's kind permission. It is not possible to give the areas beyond the year 1913, the date of the last Greenwich volume received at Kodaikánal, since no estimate is made of the areas at Kodaikánal.

Table II. is exactly similar to Table I., areas in millionths of the sun's visible hemisphere being substituted for numbers, and the percentages in the last column computed in the same manner as for Table I. The areas for years near minimum are somewhat unreliable: a large spot breaking out near the central meridian may cause a large western excess in such a year.

TABLE II.

Analysis of Areas of Sun-spots for the Years 1902–1913.

Year.	NE.	NW.	SE.	SW.	NE+SE.	NW+SW.	Total.	E–W.	Percentag
1902	5479	9020	3616	3601	9095	12,621	21,716	−3526	41·88
1903	23,622	24,603	45,605	31,748	69,227	56,351	125,578	+12,876	55·12
1904	51,290	46,854	40,054	42,963	91,344	89,817	181,161	+1527	50·42
1905	141,522	131,382	92,794	70,449	234,316	201,831	436,147	+32,485	53·72
1906	96,907	90,617	38,153	46,100	135,060	136,717	271,777	−1657	49·69
1907	89,172	88,116	101,986	112,129	191,158	200,245	391,403	−9087	48·84
1908	56,552	58,704	70,909	75,441	127,461	134,145	261,606	−6684	48·72
1909	57,797	58,274	66,948	78,691	124,745	136,965	261,710	−12,220	47·67
1910	13,713	13,776	40,768	36,237	54,481	50,013	104,494	+4468	52·14
1911	4358	1874	10,406	6696	14,764	8570	23,334	+6194	63·27
1912	13	484	4747	8600	4760	9084	13,844	−4324	35·10
1913	1215	886	243	665	1458	1551	3009	−93	48·45
1902–1913	541,640	524,590	516,229	513,320	1,057,869	1,037,910	2,095,779	+19,959	50·48

7. It will be seen from Table II. that the eastern percentage is less marked in the case of the areas than it is for the numbers; in fact, if the year 1905 were omitted, the excess would be on the other side, and only five out of the twelve years show an eastern excess. Mrs. Maunder's figures for areas give an eastern percentage of 50·68, so that the eastern percentage both in numbers and areas is about two-thirds as great during the period 1902–1913 as during the period 1889–1901.

8. It will also be noted that the year 1908 is no longer abnormal; all the other years which gave an eastern excess in numbers yield an eastern excess in areas, but the years which gave a practical equality in numbers give a distinct western excess in areas.

The facts displayed in the last column but one of Tables I. and

II. may be summed up by saying that when the sun-spot curve is rising or falling rapidly there is an eastern excess both of numbers and areas, while at maximum and minimum there is an equality of numbers but a western excess of areas.

These results must, of course, be considered as provisional only, until confirmed or otherwise by results spread over a longer period.

9. In Table III. the results of Tables I. and II. are compared with corresponding results for prominences. Columns 2 and 3 are the last columns of Tables I. and II. respectively; column 4 gives the percentages of prominence numbers observed at Kodaikánal,* while the 5th column gives the numbers from Mr. Evershed's Kenley observations for the years 1902–1905 and the areas from Kodaikánal photographs for the years 1913–1917.

TABLE III.

Comparison of Eastern Percentage of Sun-spots and Prominences.

Year.	Sun-spots.		Prominences.	
	Nos.	Areas.	Kodaikánal. Nos.	Kenley. Nos.
1902	47·92	41·88		53·64
1903	53·72	55·12		53·46
1904	52·33	50·42	50·07	51·18
1905	50·79	53·72	52·46	48·72
1906	49·84	49·69	53·25	
1907	50·00	48·84	52·61	
1908	52·66	48·72	52·43	
1909	49·93	47·67	52·64	
1910	56·02	52·14	53·92	
1911	57·35	63·27	53·00	
1912	50·3	35·10	51·53	Kodaikánal Areas.
1913	49·25	48·45	50·53	51·31
1914	49·83		49·85	51·47
1915	51·13		50·12	49·35
1916	52·00		50·51	50·63
1917	50·00		49·45	49·65

10. A careful comparison of columns 2 and 4 shows that the table can be divided into two parts between the years 1909–1910. There are two reasons for making this division. Up to 1909 the eastern excess is much more marked for prominences than for sun-spots, while after 1909 the sun-spots show a more marked eastern excess than the prominences (the apparent considerable western excess in 1913 is not real, as will be seen on reference to Table I.). In the second place, from 1910 onwards there is a very good agreement between columns 2 and 4 year by year. For example, 1910, 1911 show an unusually large eastern excess of both sun-spots and

* *Kodaikánal Bulletin*, No. 28, and later Bulletins.

prominences, while 1914 shows a small western excess in both cases. On the other hand, prior to 1910 there is very little trace of agreement between the two columns.

11. Mr. Evershed suggests that this want of agreement in the earlier years may possibly be due to the fact that prior to 1908 the results depend largely on visual observations which may be subject to systematic bias in favour of the east limb, as recorded in *Kodaikánal Bulletin*, xxviii. p. 510. From the year 1908 the visual records will have a decreasing effect on the results, and after 1914 no effect, since visual records were discontinued in that year.

Without laying too much stress on this possible explanation, we may note that it would explain the fact that during these earlier years the prominences show a greater eastern excess than the spots, and also supplies a possible explanation of the want of agreement between columns 2 and 4 prior to 1910.

12. The points on which I would lay special stress may be summarised as follows:—

(i.) During the years 1902–1917 there was an eastern excess of sun-spots both as regards area and numbers, but this excess was only about two-thirds the amount found by Mrs. Maunder for the period 1889–1901.

(ii.) This eastern excess only manifests itself in certain years, apparently only in those years when the solar activity is increasing or decreasing rapidly. Maximum and minimum years show no eastern excess in number, but show a western excess in area.

(iii.) The facts stated under (ii.), if confirmed by similar results from previous years, seem to negative the possibility of the phenomenon being due to instrumental or observational causes, including such a cause as that suggested by a writer in *Nature*.*

(iv.) From the year 1910 onwards there is a good agreement between the eastern excess of sun-spots and the eastern excess of prominences, being less marked, however, in the case of the prominences. Prior to 1910 there is no such agreement, and the prominences show the more marked eastern excess; this is, however, possibly due to observational causes. It seems natural, therefore, to assume that the eastern excess in both cases is due to the same cause.

13. In conclusion, I must express my indebtedness to Mr. Evershed, not only for calling my attention to the desirability of continuing Mrs. Maunder's work now that another sun-spot cycle has elapsed, and for the sake of comparison with the prominence records, but I am also indebted to him for allowing me the necessary facilities for carrying out the investigation, and for the interest he has taken in it.

Kodaikánal,
1918 *May* 15.

* *Nature*, 1918 Jan. 31. The suggestion put forward by J. L. does not seem to me in any way adequate to explain the phenomenon, but I refrain from repeating my arguments here, as I have already stated them in *Nature*.

Proper Motions of Stars in the Cluster M 41 (*N.G.C.* 2287).

By T. P. Bhaskaran, M.A.

(*Communicated by R. J. Pocock.*)

The Cluster M 41 (Canis Major) lies in the region undertaken by the Nizamiah Observatory for the Astrographic Catalogue. A photograph of the region was taken and measured during the usual course of the work. It was found that four photographs of the Cluster had been previously obtained at Cordoba, and the measures published by Gould in his *Cordoba Photographs*. As the interval between the Hyderabad and the Cordoba plates is thirty-five years, it was thought that a comparison between the two might be interesting. The Cordoba places were accordingly converted into "standard co-ordinates," adopting the Hyderabad plate centre, and then directly compared with the Hyderabad measures by applying a linear correction. The number of stars compared was 136. No sensible proper motion, however, could be detected with the rather large unit of measurement adopted for the Astrographic Catalogue, except in the case of six stars (which presumably do not belong to the Cluster proper): the other differences (Hyderabad–Cordoba) are always small.

The mean difference without regard to sign are :—

Δx.	Δy.
″	″
0·36	0·42

The cases of large differences (H–C) are :—

Cordoba.		Hyderabad.			Δx.	Δy.	Centennial
No.	Mag.	No.*		Mag.	Reseau int.	Reseau int.	P.M.
							″
3	9¼	−20°	15309	10·6	+·008	+·009	10·3
19	10	−20°	15581	11·7	+·005	−·002	4·6
105	9¾	−20°	15254	11·7	+·005	+·001	4·4
116	8½	−20°	15553*	10·2	·000	−·005	4·3
127	9¾	−20°	15554	11·1	−·002	+·005	4·5
143	10	−20°	15692	11·6	+·003	−·013	11·4

(The plus sign in the column Δy denotes that the star is moving south.)

The difference of epoch is 35·0 years in each case.

One star, No. 1 in the Cordoba volume, is not to be found on the Hyderabad plate, though the latter shows stars down to the 13th magnitude: neither is it measured in the later Cordoba plates.

* These measures are not yet in print: the numbers are taken from the manuscript copy for press.

In the Catalogue of Clusters on the Franklin-Adams photographs of the sky (see *Memoirs R.A.S.*, 60, part v.) this is described as a large open Cluster, 30′ diameter, but is erroneously designated M 14, following the N.G.C. (the N.G.C. error is corrected in *Memoirs R.A.S.*, 51, 225).

Nizamiah Observatory, Hyderabad:
1918 *Aug.* 25.

The Five Aberrations of Lens-Systems.

By A. E. Conrady.

In 1856 Seidel published a classical paper (*Ast. Nach.*, 43, Nos. 1027–1029) in which he extended the Gaussian theory of centred systems of spherical surfaces so as to include all aberrations of the third order, which had been treated as negligible by Gauss. As the result of this work Seidel proved that the images produced by such systems were subject to five, and only five, distinct forms of aberration, viz. ordinary spherical aberration, coma, astigmatism, curvature of the field, and distortion, of which the last four are peculiar to extra-axial points of the image.

Seidel's treatment of the problem was thorough and exhaustive, but decidedly lengthy and complicated. As a consequence the value of his work has only in recent times been widely recognised; but even now one hesitates to include it in the curriculum of ordinary students of practical optics.

Recently I suddenly realised that all the essential features of Seidel's theory, and in fact some additional deductions of considerable value which escaped him, can be derived in an extremely simple way from the well-known law of spherical aberration for points on the optical axis: that the lateral deviation of a marginal ray at the paraxial focus is proportional to the third power of the aperture.

In fig. 1 let AP represent the axial section of the first refracting surface of a centred system, C the centre of curvature of the surface, O the axial point of an object, and F its image, by paraxial rays; then the law of spherical aberration states that a ray starting from O at a finite but small angle with the optical axis will cut the paraxial focal plane at a distance $FG = k.y^3$, where k is a constant depending only on the radius of curvature and on the two conjugate distances, and that FG will lie in the plane containing the optical axis and the point of incidence of the ray. Now let us consider an extra-axial point O_1 of the object, and let us assume for the present that the latter lies on a sphere having its centre at C. If we draw the "auxiliary optical axis" O_1C which penetrates the refracting surface at A_1, we shall have precisely the same relations between O_1 and this auxiliary axis as between O and the principal optical axis: hence the focus of a thin "radial"

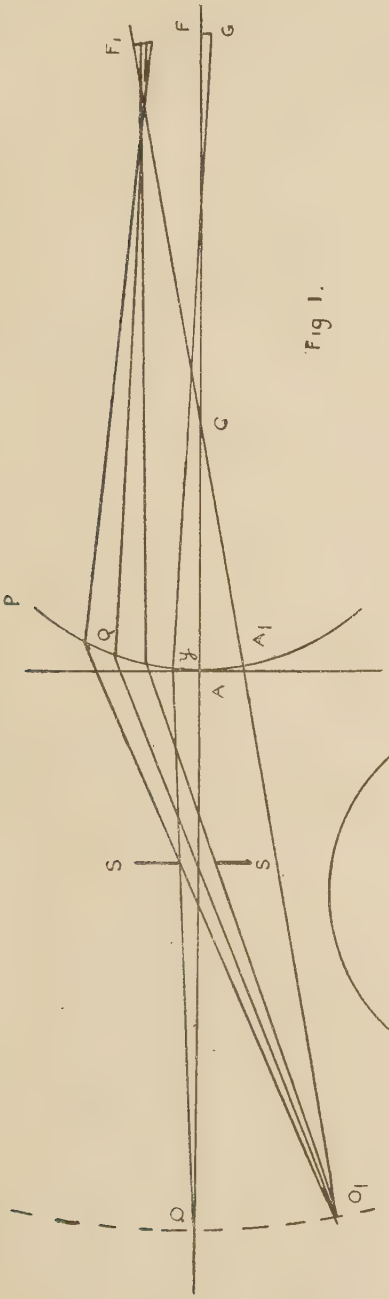


Fig 1.

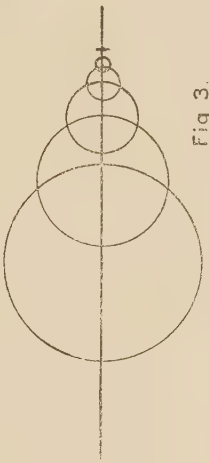


Fig 3.

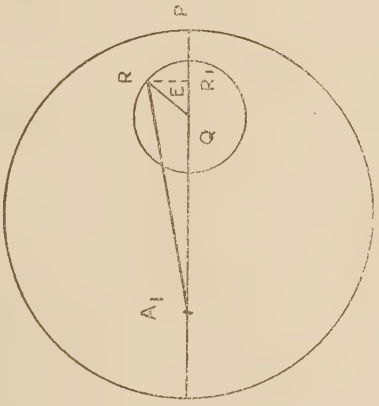


Fig 2.

pencil surrounding O_1C will be formed at a point F_1 of O_1C, produced so that $CF_1 = CF$, and we shall have the same aberration-constant k for rays at finite angles with O_1C as for those referred to OC.

Now let SS represent a stop or diaphragm concentric with the optical axis of the system which limits and regulates the pencils admitted to the system. This diaphragm will in general cause a more or less eccentric pencil to pass from O_1 to the refracting surface and thence towards the focus F_1. This oblique pencil will cut a plane touching the refracting surface at its axial pole A in a precise circle. Strictly speaking, its intersection with the refracting surface itself will be distorted, but under the limitations to small angles and curvatures, the amount of the distortion will be limited to small quantities of the third order and could only affect the aberrations of fifth and higher orders which we are neglecting; and the same consideration will apply to the projection of the intersection of our oblique pencil with the refracting surface upon a plane touching the latter at A_1. We now consider this latter projection of fig. 1, *i.e.* a view of it in the direction F_1C of our auxiliary optical axis (fig. 2). We then have A_1P as the trace of the plane containing the principal optical axis and the extra-axial object point, Q, as the point where the central or "principal" ray of our oblique pencil cuts the refracting surface, and a circle of radius y as the outline of the pencil. If we take any point R on the circumference of this circle, the ray passing through it will have a lateral spherical aberration in the plane of F_1 equal to $k \cdot \overline{A_1R}^3$, and this will be in the reverse direction of A_1R if we assume spherical under-correction, which is usually taken as the positive sense of spherical aberration. We now resolve this displacement of our ray from its ideal coincidence with F_1 into components respectively parallel and perpendicular to the plane of the optical axis and easily find

$$\text{Horizontal component} = k\overline{A_1R}^3 \cdot \frac{A_1R'}{\overline{A_1R}} = k\overline{A_1R}^2 \cdot A_1R'$$

$$\text{Vertical component} = k\overline{A_1R}^3 \cdot \frac{RR'}{\overline{A_1R}} = k\overline{A_1R}^2 \cdot RR'$$

Introducing the symbols Y for the distance A_1Q and E for the angle PQR, we next have by inspection of our diagram

$$A_1R' = Y + y\cos E \qquad RR' = y\sin E$$

$$\overline{A_1R}^2 = (Y + y\cos E)^2 + y^2\sin^2 E = Y^2 + 2Yy\cos E + y^2$$

and introducing these we find the components of the lateral aberration

$$\text{Horiz. comp.} = (Y^3 + 3Y^2y\cos E + Yy^2(1 + 2\cos^2 E) + y^3\cos E)k$$

$$\text{Vert. comp.} = (Y^2y\sin E + 2Yy^2\sin E\cos E + y^3\sin E)k$$

These equations display Seidel's five aberrations practically in the form in which he obtained them; but the last but one

term of each is capable of a modification which renders its significance clearer. By adding $1-\cos^2 E-\sin^2 E=0$ to the coefficient $(1+2\cos^2 E)$ in the horizontal component, it becomes $2+\cos^2 E-\sin^2 E=2+\cos 2E$, whilst in the vertical component we can introduce $2\sin E\cos E=\sin 2E$; the equations then take the form

$$\text{Horiz. comp.} = (Y^3+3Y^2y\cos E+Yy^2(2+\cos 2E)+y^3\cos E)k$$
$$\text{Vert. comp.} = (Y^2y\sin E+Yy^2\sin 2E+y^3\sin E)k$$

But before discussing their significance we must remove the restriction to only one refracting surface and to the special position of the extra-axial object point on a sphere concentric with that surface.

If the object lies on a surface of any other finite curvature, or on a plane, the distance O_1A_1 will be altered by a small quantity of the second order; the distance A_1F_1 of its paraxial image will also be altered by a corresponding small quantity of the second order; but this will not sensibly alter the aberration-constant k for the surface, as it would only be affected by finite changes of the conjugate distances. No difficulty arises therefore from this generalisation.

The angle E which measures the location of a ray on the circumference of the pencil remains unchanged (within small quantities of the second order) from surface to surface for the reasons which were given with reference to the sensibly unaltered circular outline of the oblique pencil in different projections at small angles with each other.

The radius y of the oblique pencil will vary from surface to surface according to the separation between them and the convergence or divergence of the rays; but at each surface it will be in some fixed proportion to the radius S of the original stop SS (fig. 1); by putting $y_\gamma=c_\gamma . S$ we can therefore express all the y in terms of S.

For the second and subsequent surfaces the image produced by the first or preceding surface will take the place of the object, and the auxiliary optical axis defined by this object and the new centre of curvature may take all kinds of small inclinations to the principal optical axis, and the values of the Y will vary accordingly. But it is easily seen that so long as all the angles are small, the Y at any one surface will grow in exact proportion (within small quantities of the third order) with the angle V which the original extra-axial object-point subtends at the centre of the stop, *i.e.* with the semi-angle of the field of view. Hence we may put $Y_\gamma=d_\gamma . V$, where d_γ is another factor which is constant for any one surface, but varies widely for different surfaces.

We thus arrive at the general equations for the two components of the lateral aberration arising at the γ^{th} surface

$$\text{Horiz. comp.} = (d_\gamma^3V^3+3d_\gamma^2c_\gamma V^2S\cos E+d_\gamma c_\gamma^2VS^2(2+\cos 2E)+c_\gamma^3S^3\cos E)k_\gamma$$
$$\text{Vert. comp.} = (d_\gamma^2c_\gamma V^2S\sin E+d_\gamma c_\gamma^2VS^2\sin 2E+c_\gamma^3S^3\sin E)k_\gamma.$$

It now only remains to determine how the total aberration existing at one surface combines with that arising at the next. Each surface produces a magnified or diminished image of the image presented to it by the previous surface, and this image is only subject to a distortion (represented, as we shall see, by the term in V^3), which is a small quantity of the third order. As the whole aberration-displacement of any ray is itself only a small quantity of the third order, the small distortion will be insensible in the image of so small an object and the displacement will therefore reappear in the new image simply magnified or diminished, but unchanged in its direction. Hence both components will retain their analytical form, however many surfaces there may be, and the final aberration may be written in the form

$$\text{Horiz. comp.} = m_1V^3 + 3m_2V^2S\cos E + m_3VS^2(2+\cos 2E) + m_4S^3\cos E$$
$$\text{Vert. comp.} = \qquad m_2V^2S\sin E + m_3VS^2\sin 2E \qquad + m_4S^3\sin E$$

in which m_1 m_2 m_3 and m_4 are constants which may have any positive or negative value, and which will be more and more independent of each other the greater the number of surfaces, primarily on account of the great variability of the value of the coefficient d_γ at successive surfaces.

The terms of our two final equations taken from right to left represent the five aberrations in the order adopted by Seidel. We will discuss them separately, each one as if the others did not exist or had been reduced to zero.

The two terms $m_4S^3\cos E$ and $m_4S^3\sin E$ when combined and applied to all values of E from 0 to 360°, represent a circle of radius m_4S^3 around the final image of the original object-point O, which would be produced by successive radial thin pencils. The marginal rays admitted by our stop SS would therefore be equally distributed around this circle, and indicate that they have a focus either shorter (if m_4 is positive) or longer (m_4 negative) than that produced by the radial pencils. This is ordinary spherical aberration, and the important conclusion (not usually emphasised in discussions of Seidel's analysis) is that these terms do not depend in any way on V, and therefore that the spherical aberration of any lens-system is, in first approximation, constant over the whole field. If it is corrected on the optical axis, it is also absent in the oblique pencils. This is of immense value in shortening the exact computation of lens-systems.

Coming to the terms in VS^2, we find the angle 2E in both components; therefore these components have the same value for E and for 180° + E, which means that a ray from any point on the circumference of the stop meets the one from the opposite point in our adopted focal surface.—For E = 0 and E = 180° the horizontal component (which will lie in the plane of the optical axis, because the vertical component is zero for these values of E) reaches its maximum value $= 3m_3VS^2$, and for the principal ray through the centre of the stop it reaches its minimum zero.

because S is then zero. This justifies the practical computer's practice of tracing these three rays in the plane of the optical axis by his simple formulæ and estimating the *Coma* by the distance of the intersection of the two marginal rays from the principal ray, for this is the maximum dimension of the Coma-figure. The latter is at once revealed when we discuss the two components for all possible values of E and S, for this shows that the rays for any particular diameter of the stop are distributed in pairs, round a circle of radius $m_3 VS^2$ with its centre at a distance $2m_3 VS^2$ from our reference-point. The totality of these circles corresponding to successive concentric zones of the diaphragm-opening produces the well-known comet-like shape (fig. 3).

In the discussion of the terms in V^2S we reap special advantages from our otherwise artificial reference of the phenomena to the final image which would result from successive images produced by thin radial pencils. It is well known (Seidel mentions it in a footnote) that this procedure leads to a final image which has the curvature defined by the Petzval-formula; hence the two astigmatic foci which are expressed in the terms in V^2S are referred to the Petzval-curvature as a reference-surface. Now the discussion of these terms in the manner already applied to the spherical aberration terms readily shows that in our reference-surface the rays from the circumference of the stop will be distributed over the circumference of an ellipse with a horizontal axis equal to three times its vertical axis, that the term in the horizontal component corresponds to the focal line perpendicular to the plane of the optical axis, and the term in the vertical component to the focal line in the plane of the optical axis, and that *the former always lies three times as far from the Petzval-curve as the latter*, but on the same side of it. This invariable relation between the Petzval-curvature and that of the two astigmatic focal surfaces seems to have escaped the attention of Seidel as well as of his successors in this branch of analytical optics, probably because all referred the phenomena to the paraxial focal *plane* and so obtained more complicated expressions for the astigmatic terms. This constant ratio is another fact of great value to practical computers, because the Petzval-curvature can be most easily computed by the well-known simple formula; and next in ease of computation comes that of a fan of rays in the plane of the optical axis which reveals the location of the vertical focal line or the "tangential focus." By the law of the three-to-one relation we can then, without any separate calculation, locate the horizontal focal line or "sagittal focus" as lying between the two others, at one-third of their separation from the Petzval-curve. As special applications of this law we can predict that if the field of a system is made flat for tangential rays (a favourite state of correction) the curvature of the field of the sagittal rays will be two-thirds of the Petzval-curvature and that the other special correction frequently found in photographic objectives, when the two astigmatic surfaces have equal but opposite curvature, must make the curvature of both

numerically equal to half the Petzval-curvature. It also follows that if the astigmatism is corrected, the stigmatic image must necessarily have the Petzval-curvature. In the light of these results the Petzval theorem acquires a much wider significance than has ever been attributed to it.

The only remaining term of our equations is that in V^3, which occurs only in the horizontal component. It depends on the angle of the field only, and is thus present in undiminished magnitude, however small the diaphragm opening may be. It represents distortion, but it must be stated that with regard to this term the equation is not exhaustive, because it is based on the *curved* images produced by the radial pencils. This curvature of the intermediate images can easily be shown to lead to additional terms of the V^3 order, which would have to be included to secure a numerically correct value of the distortion.

It is, however, hardly worth while to develop these terms, for I am strongly of opinion that the proper use of approximate analytical solutions of optical problems consists in deriving general conclusions from them, and not in working out numerical results, for the latter rarely are sufficiently accurate to be of practical utility.

I should like to add that the most characteristic of the fifth order aberrations are displayed with the same clearness and facility as the third order ones if the lateral aberration is put $= k_1y^3 + k_2y^5$ and the resulting expression resolved into components exactly as for the third order aberration only. A complete discussion of the fifth order would, however, lead to great complications, because the various simplifying assumptions used in the paper are no longer admissible when the fifth order terms are included.

Imperial College,
1918 *Nov.* 7.

Differential Transit Observations. By W. E. Cooke, M.A.

At the Sydney Observatory we are just commencing transit observations of the "Intermediate Stars" in the zones 52°–65° South Declination; and in aiming at the greatest possible efficiency I have introduced the following features:—

Mechanical.

The scheme is arranged for two observers, one (A) recording transits, and the other (B) reading the circle. An effort has been made to afford all possible comfort, and to avoid the necessity for either observer to move from his position during the evening; also to eliminate all unnecessary observations. In other words, to secure the maximum output and accuracy with the minimum effort.

The Observing Couch.—A lies down on an upholstered couch, with or without a small pillow under his head, according to taste,

and covered with a rug in cold weather. The couch is mounted on four bicycle wheels, which run along the meridian on the reverser rails. By handling these wheels in a manner similar to that of an invalid chair, A can easily propel himself backwards and forwards. The couch is hinged in the middle, and the portion supporting the observer's back and head can be raised or lowered through any arc up to a whole quadrant by means of a small electric motor. Thus, by the pressing of a button and a slight pull on the wheel, A can bring his eye to the exact place for comfortable observing. The total range in declination being 15°, a small movement only is necessary, and this can be effected in a few seconds and with the slightest possible effort.

The Eyepiece End.—In observing, A has choice of a transit micrometer or a set of seven fixed wires. If the star is sufficiently bright, the moving wire is used, but if the star is so faint as to cause eye-strain in following it, the older method is adopted. This, of course, makes it necessary to investigate the relation between the two methods and apply two different collimation errors, and to investigate the magnitude equations, especially for the faintest stars.

Setting the Telescope.—A can easily, without change of position, set the telescope approximately to any given declination by means of the following simple device: Due south of the centre of the telescope axis, just clear of the line of collimation, an optical lantern throws a spot of light, intersected by a horizontal wire, upon a little strip of mirror fixed to the central tube of the telescope. Thence it is reflected south again, and is seen upon a vertical scale underneath the lantern. This scale is made very simply of a strip of wood about five feet long and rather more than an inch in width, covered with drawing-paper; and the degree lines and numbers are drawn boldly upon it with Indian ink. It is mounted vertically from the floor about six feet due south of the telescope, and each degree-space is about $2\frac{1}{8}$ inches. The spot of light spreads so as to illuminate two or three degrees, and the observer A can, by the slightest movement of his head, read off the approximate pointing on the telescope with ease, or can move the telescope to any given setting. In practice this is the first step in an observation. At the completion of the previous observation B calls out the setting for the next star. A moves the telescope to the approximate place and clamps, and B completes the pointing by means of a handle which gears into the slow-motion screw. The telescope is then unclamped, in order to avoid mechanical strain, and the final setting in declination is made by A with the micrometer wire.

A Mechanical Working Catalogue.—This consists of twenty-four strips of linen-mounted drawing-paper, each one representing 1^h of R.A., with 6^m overlap at the ends. They are 16×4 inches, ruled with seventy-two lines, each line representing 1^m of R.A. Upon these the star positions are graphed, and any required information, such as magnitude, pointer setting, etc., inscribed. One of these strips, corresponding to the hour of observation, is wrapped round a brass cylinder attached to a synchronome clock movement

(second by second), and set so that the correct sidereal time is marked by a fixed straight-edge parallel to the axis of the cylinder. This straight-edge thus represents the meridian, and a glance shows the star or stars that are just "coming on" at any moment. Several cylinders are provided, and a sufficient number loaded ready for use before the evening's work commences. The cylinder rotates once in 72^m, thus allowing B a choice of any moment within 12^m round about the even sidereal hour in which to change cylinders.

Observation and Computation.

The observations are limited to stars within the 15° (50°–65° dec.), and special observations for level and azimuth errors are eliminated. All necessary instrumental errors (except collimation) are deduced in a manner analogous to that employed in the computation of astrographic plate constants.

This step is not taken primarily with the view of shortening the work, although it actually effects that object, but in accordance with the principles advocated in my paper published in *Monthly Notices*, **72**, 152.

Tables of differential star corrections for these zones have been computed according to the formulæ given in *Monthly Notices*, **77**, 465.

The computations are carried out according to the following scheme:—Let $c_1 + r$ be the star correction, apparent to mean, where r is taken from these tables and c_1 is an unknown constant for the evening. $c_2 + ax\ (= m + n \tan \delta) =$ the correction for level and azimuth errors, where c_2 and x are unknown, and $a = \tan \delta$.

$c_3 + ty =$ clock correction, where c_3 is the unknown error of the clock at some whole sidereal hour (T) prior to the commencement of observations, t is the time elapsed since T, expressed in decimals of an hour, and y is the hourly clock-rate.

Now let A represent the mean of wires, corrected for collimation error, diurnal aberration, proper motion, and differential star correction (r), and B represent the tabular position of a fundamental star at the beginning of the year, then

$$\begin{aligned} \mathrm{B} - \mathrm{A} &= c_1 + c_2 + ax + c_3 + ty \\ &= ax + ty + z, \text{ say.} \end{aligned}$$

The solution of these formulæ for the fundamental stars gives values of x, y, and z for the evening. Two small tables are then formed, combining differential star corrections, x, y, and z, under arguments α and δ respectively, and the list stars are reduced straightaway, with two corrections only, from "mean of wires collimation" to "mean R.A. Jan. 1."

It may perhaps be objected that the clock-rate is sufficiently well known, and so the above process can be further simplified. Of course a little time might be saved by this assumption, but what transit observer has not had reason to doubt the correctness of the average daily clock-rate, when applied to any given period

of 2 or 3 hours? Not perhaps that the clock-rate itself varies to any extent—though even that is not certain—but that the *effective* clock-rate does. Whether it is due to a gradual fatigue or some personal characteristic I know not, but when working up clock errors from even Nautical Almanac stars in the ordinary direct method, it sometimes happens that the apparent clock-rate for a few hours differs quite noticeably from the daily average. So I consider we shall obtain more accurate results by introducing the *effective* rate as an unknown into the equations of condition, and computing it directly from the observations themselves, provided the time period is sufficiently long.*

Analogous methods are used for the N.P.D. observations. Corrections for differential refraction have been tabulated. The main table gives the *difference* between the refraction (at $30^{in.}\cdot0$ and 60° F.) for 15° Z.D. (the upper limit of the Zones) and any other position down to 31° Z.D., with pointer reading as argument. Subsidiary tables give small corrections for barometer and temperature, which are, for convenience, worked in to the daily correction table for differential reduction from apparent to mean N.P.D. Practically, therefore, the correction for refraction consists of a single quantity taken by inspection directly from a table. After applying the corrections to the corrected circle reading of each fundamental star for differential refraction and differential reduction (and proper motion) the result is subtracted from the tabular N.P.D. on Jan. 1; a mean of these results is taken, and applied as a constant correction to each star. The result is the observed N.P.D reduced to Jan. 1.

It may perhaps be as well to add that a test of the new differential reductions in both R.A. and N.P.D. was made by comparing a number of computations made with these tables and Turner's respectively. For this purpose some of the old transit reductions were used, the stars being selected at random. The new differential tables are constructed to only two places of decimals in R.A. and one in N.P.D., and the individual results obtained by simple interpolation, so errors of the order of $0^{s}\cdot01$ might be introduced on this account. The differences between the two methods are thus summarised:—

In R.A. difference of	$0^{s}\cdot00$	= 58	stars.
,,	$\cdot01$	= 71	,,
,,	$\cdot02$	= 7	,,
In N.P.D ,,	$0''\cdot0$	= 111	,,
,,	$0\cdot1$	= 25	,,
,,	$0\cdot2$	= 0	,,

* Several months have now elapsed since observations commenced, and the results show very little variation, as a rule, between the clock-rates over the period of observation, as above deduced, and the average daily rate. The computations, therefore, have been simplified to the extent of adopting the average rate for y, except on the few occasions that actually do occur, when the effective value of y has undoubtedly changed. In these cases y is computed as already indicated.

The Energy of Magnetic Storms. By S. Chapman, M.A., D.Sc.

1. Magnetic storms are due primarily to electric currents flowing in high layers of the earth's atmosphere, though these are accompanied by induced currents within the earth. The magnetic variations produced by the currents are partly local, irregular, and rapidly changing, but a fundamental characteristic of magnetic storms is a world-wide diminution in the horizontal intensity of the earth's magnetic field near the surface. There are, indeed, three principal "world-wide" phases: (I.) a rapid initial increase in horizontal force, lasting for a fraction of an hour; (II.) a much larger diminution lasting for many hours; and (III.) a slow return to the normal magnetic field, taking some days to accomplish. Associated with these changes in the horizontal field there are corresponding but minor variations in the vertical component of magnetic force. The variations in these two elements during the first forty-eight hours of a moderate storm, for stations in the tropics, are represented in fig. 1.*

2. The initial phase will not be considered in this paper. I hope on a later occasion to describe the results of a special study of the phenomena which occur at the commencement of a magnetic storm, and among other questions that of the related energy will be dealt with. It is here sufficient to remark that this energy is a very small fraction of the whole amount involved in a storm.

3. The second and third phases may be respectively termed the "forced" phase and the phase of recovery (or decay). The former, as above defined, ends at the time of minimum in the mean horizontal intensity round the equator (say), but the causes which are acting during phase II. do not cease at this minimum epoch. They are relaxed only gradually, as is clearly shown by the above curve representing the variation of horizontal force; not till some time after the minimum epoch does the variation assume the exponential type corresponding to a free decay of the current system and the associated magnetic field.†

4. During the storm the total energy of the magnetic field in and around the earth is increased, but finally it returns to its original value. At some stage the excess energy E of the field will attain a maximum value E_0. During the decay of the disturbance field this energy is withdrawn and dissipated in the form of heat in the atmospheric and internal strata wherein the electric currents circulate; the greater the resistance of these strata, the more rapidly is the energy dissipated.

* This is drawn from the left-hand section of fig. 1 of my recent "Outline of a Theory of Magnetic Storms," *Proc. Roy. Soc.*, A, 95, 61, 1918; *cf.* also Moos, *Colaba Magnetic Observations*, 1846–1905, 2, ch. 10, 1910.

† The minimum epoch is usually more rapidly attained in an intense than during a moderate storm (*cf.* fig. 6 of the paper just cited). This seems to denote that the streams of corpuscles to which, in that paper, the production of the storm is attributed, are narrower the greater their intensity. In weak storms the passage through minimum epoch is very slow, a steady state being arrived at and maintained almost unaltered for many hours.

But this is not the only energy so transformed in the current strata. During the production and increase of the current system, and during the period of gradual relaxation before free decay commences, energy must continually be afforded by the mechanism which generates the electromotive forces, in order to supply the continual loss in overcoming the electrical resistance. The electromotive forces, in fact, do double work during the forced phase of the storm. They supply the heat-loss in overcoming resistance, and also the energy of the magnetic field. The latter suffices to maintain the currents for some time after the primary electromotive forces have ceased to act, *i.e.* during the phase of decay.

5. The forced phase and period of gradual (not *free*) relaxation

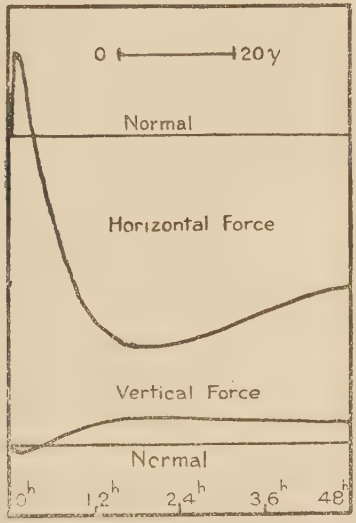


Fig. 1.

occupy less time than the period of decay, but the mean current intensity during the former is greater than that during the latter interval. An exact calculation of the ratio between the amounts of energy lost as heat during the two periods is hardly possible at present, but it seems unlikely that the two amounts will be very unequal. Now the energy loss during the period of decay is known to be approximately equal to E_0, the maximum excess energy of the magnetic field during the storm. Hence the total energy to be supplied by the external electromotive forces in the course of the storm may be roughly estimated as $2E_0$; and this takes into account all the energy directly concerned in the magnetic disturbance, whether related to the atmospheric currents or to those within the earth. The energy is ultimately transformed to heat in the current-conducting layers.

6. The energy of a storm is thus measured in terms of the maximum excess energy appearing in the magnetic field during the storm. It will be shown that the excess energy E attains its maximum E_0 at the epoch of minimum mean horizontal intensity (*cf.* fig. 1), and that it depends mainly on δH_0, the maximum mean *diminution* of horizontal intensity along the equator. The value of E_0 is, in fact, nearly proportional to δH_0, and not to the *square* of the intensity of the disturbance, as has generally been supposed.

7. The disturbance magnetic field will be supposed resolved into its spherical harmonic components, some of which arise from the external atmospheric current sheet (which we may regard as spherical and of radius b), and some from within the earth. The earth's radius a is $6{\cdot}4 \cdot 10^8$ cm., and as the height of the outer current sheet is probably about 100 km., b will be about $6{\cdot}5 \cdot 10^8$ cm.

The potential of a typical harmonic term of external origin may be written

$$B_n b^n S_n^m / r^{n+1} \quad . \quad . \quad . \quad . \quad (1)$$

outside the current sheet, and

$$- B_n r^n S_n^m / b^{n+1} \quad . \quad . \quad . \quad . \quad (2)$$

inside it. Here r is the distance from the centre of the earth, B_n is a constant, and S_n^m is a surface harmonic of degree n and order m, *i.e.* it is a function of θ the colatitude, and ϕ the longitude, of the form

$$P_n^m(\cos\theta)\cos(m\phi + \epsilon) \quad . \quad . \quad . \quad (3)$$

where $P_n^m(\cos\theta)$ is the tesseral harmonic of degree n and order m.

8. The potential outside the earth's surface of a corresponding harmonic component arising from currents beneath the earth's surface may be written

$$A_n a^n S_n^m / r^{n+1} \quad . \quad . \quad . \quad . \quad (4)$$

where A_n is some constant. The form of the potential within the earth is unknown, but for the present approximate calculation it will be sufficient to regard the currents as flowing in a thin current sheet of radius a, *i.e.* immediately beneath the surface. A similar simplifying assumption will be made with reference to the potential of the earth's "permanent" magnetic field; in neither case is the assumption true, but the convention affects the calculations here made in a way which can be roughly gauged: and the error is found to be relatively unimportant. Hence the internal potential corresponding to (4) will be written

$$- A_n r^n S_n^m / a^{n+1} \quad . \quad . \quad . \quad . \quad (5)$$

9. Maxwell's formula for the energy of a magnetic field is the integral

$$\frac{1}{8\pi}\int\int\int(\alpha^2 + \beta^2 + \gamma^2)dxdydz \quad . \quad . \quad . \quad (6)$$

taken throughout the field; α, β, γ are the rectangular components of magnetic force at each point. Suppose that α_0, β_0, γ_0 are the components of the permanent field, and $\delta\alpha$, $\delta\beta$, $\delta\gamma$ those of the disturbance field at any time (these are always small compared with the intensity of the permanent field); then

$$\alpha = \alpha_0 + \delta\alpha,\ \beta = \beta_0 + \delta\beta,\ \gamma = \gamma_0 + \delta\gamma.$$

The excess energy of the field due to the disturbance is consequently

$$\frac{1}{8\pi}\iiint\{(\delta\alpha)^2 + (\delta\beta)^2 + (\delta\gamma)^2\}dxdydz \quad . \quad . \quad (7)$$

$$+\frac{1}{4\pi}\iiint(\alpha_0\delta\alpha + \beta_0\delta\beta + \gamma_0\delta\gamma)dxdydz \quad . \quad . \quad (8)$$

Here (7) is equal to what would be the energy of the disturbance field if existing by itself, in the absence of the permanent field. This will be called the "self-energy integral," while (8) will be termed the "joint-energy integral." The latter has usually been neglected, on the ground that, the permanent and disturbance fields being independent, on the average there will be no net gain or loss due to their superposition. This is true as regards the harmonic components corresponding to local irregular fluctuations of magnetic force during the storm, and also as regards what I have elsewhere termed the "local-time variations," which mainly represent a longitudinal inequality in the world-wide diminution of horizontal magnetic intensity (*loc. cit.*, § 1). But it is not true of the harmonic component corresponding to this latter variation, averaged along each circle of latitude, and the term hence arising in the joint-energy integral proves to be the most important part of the whole excess energy.

10. The calculation is rendered simpler if the integrals (7) and (8) are transformed by Green's theorem into surface integrals, taken over the inside and outside of each current sheet. The equivalent surface integrals for the excess energy are

$$\frac{1}{8\pi}\iint\delta\Omega\frac{d\delta\Omega}{dn}ds \quad . \quad . \quad . \quad . \quad (9)$$

$$+\frac{1}{4\pi}\iint\Omega_0\frac{d\delta\Omega}{dn}ds \quad . \quad . \quad . \quad . \quad (10)$$

where Ω_0 and $\delta\Omega$ are the potentials of the permanent and disturbance magnetic fields respectively, while the differential element dn is taken along the normal drawn *towards* the surfaces integrated over. This differentiation only affects the radial factors in the various harmonic terms of types (1) to (5), which enter into the expressions for Ω_0 and $\delta\Omega$. The surface harmonics S_n^m are therefore unaffected, and by a well-known property of these functions no contribution is made to the value of the integrals (9) and (10) by the parts arising from the product of dissimilar

harmonics. Hence in (9) each harmonic can be taken separately by itself, or, in other words, the "self-energy" of the disturbance is the sum of the self-energies of the separate harmonic components. Similarly in (10) the only terms which require evaluation are those resulting from like surface harmonics existing in the potentials Ω_0 and $\delta\Omega$ of the permanent and disturbance magnetic fields.

11. The chief surface harmonics representing the world-wide diminution of horizontal force are $P_1(\cos\theta)$, $P_2^1(\cos\theta)\cos(\phi+\epsilon_1)$, and $P_4^1(\cos\theta)\cos(\phi+\epsilon_2)$; of these the first is the most important —it represents the mean variation averaged round each circle of latitude; the other terms relate to the longitudinal inequality in the main variation. The irregular and rapidly changing magnetic variations during a storm are of generally local character, most intense round the poles, and but little felt near the equator. They can be represented only by the sum of a number of harmonic components of high degree.

12. The principal components in Ω_0 are $P_1(\cos\theta)$ and $P_1^1(\cos\theta)\cos(\phi+\epsilon)$, corresponding to a uniform magnetisation slightly oblique to the earth's geographical axis. Clearly the only *like* harmonics in the potentials Ω_0 and $\delta\Omega$, among those so far mentioned, are the terms $P_1(\cos\theta)$ occurring in each. Any other correspondences which may chance to exist will be between quite minor and insignificant components, whereas $P_1(\cos\theta)$ is the principal harmonic both in Ω_0 and $\delta\Omega$.* Hence this is the only term which need be considered in evaluating the joint-energy integral (10).

13. The calculation of this term will first be effected on the simplified hypothesis that the disturbance is of purely external origin, the earth currents and their field being neglected. The significant term in $\delta\Omega$ is then

$$\begin{cases} \delta\Pi \,.\, b^3\cos\theta/r^2 & (r>b) \quad . \quad . \quad . \quad (11) \\ -\delta H \,.\, r\cos\theta & (r<b) \quad . \quad . \quad . \quad (12) \end{cases}$$

where δH denotes the mean diminution of horizontal force round the equator. The corresponding term in Ω_0 is

$$\begin{cases} H_0 a^3\cos\theta/r^2 & (r>a) \quad . \quad . \quad . \quad (13) \\ -H_0 r\cos\theta & (r<a) \quad . \quad . \quad . \quad (14) \end{cases}$$

the last value being an approximation (§ 8). Here H_0 denotes the ordinary mean horizontal force along the equator, which is approximately 0·316 in c.g.s. measure.

The part of the joint-energy integral corresponding to the space outside the sphere b is found, from the integral over the outer surface of this current sheet, to be

$$\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left(H_0\frac{a^3}{b^3}\cos\theta\right)\left(2\delta H\,.\,\frac{b^3}{b^3}\cos\theta\right)b^2 d\phi\, d\cos\theta = \tfrac{2}{3}H_0\delta H\,.\,a^3 \quad (15)$$

* The theory referred to in the footnote to § 1 offers a simple explanation of this fact.

which is positive and, somewhat curiously, independent of b. The integrals over the inside of the sphere b and the outside of the sphere a are equal in magnitude ($\frac{1}{3}H_0\delta H \,.\, a^3$) but opposite in sign, and their sum, which represents the part of the volume integral (8) arising from the space between the earth's surface and the atmospheric current layer, is consequently zero. The integral over the inside of the sphere a, representing the energy integral taken throughout the earth's interior, is

$$\tfrac{1}{3}H_0\delta H \,.\, a^3 \quad . \quad . \quad . \quad . \quad (16)$$

The latter is an approximation, on account of the assumption made as to the form of (14) giving Ω within the earth. This supposes that the earth's magnetism has its source near the surface, while more probably the source is distributed throughout the interior. But if, then, the magnetisation be supposed (for mathematical convenience) due to an internal current sheet of radius a_1, the surface value of the equatorial horizontal force being H_1, we shall have $H_1a_1^3 = H_0a_0^3$; hence (16) is probably fairly near the truth. Hence on adding together (15) and (16) we obtain the expression

$$H_0\delta H \,.\, a^3 \quad . \quad . \quad . \quad . \quad (17)$$

for the total excess energy arising from the joint integral (10).

14. The sign of the joint-energy integral is positive, representing an increase in the energy of the magnetic field. This may seem curious, considering that the most notable effect of magnetic storms is to diminish the horizontal magnetic intensity. The above value of $\delta\Omega$, however, corresponds to an *increase* in the magnetic field within the earth and outside the atmospheric current sheet. Only between the two is there anywhere a diminution of field intensity, and that only near the equator; for while the horizontal force is diminished, the vertical force is increased, so that near the poles the energy of the field is increased. As has been seen in § 13, the two effects just balance, and the total energy of the field between the earth's surface and the current sheet is unaffected. Fig. 2, which shows a few typical lines of magnetic force corresponding to the potential functions Ω_0 and $\delta\Omega$, will illustrate these remarks. The two circles represent the two supposed current sheets, and the thick and thin dotted lines respectively indicate (in general form) the lines of magnetic force corresponding to Ω_0 and $\delta\Omega$.

The increased intensity of force within the earth during a magnetic storm is of some interest. It always acts approximately in the same direction, and is brought into play at frequent intervals. It should therefore tend to increase the magnetisation of any parts of the earth's substance which are susceptible to magnetic influence; by thus modifying that portion of the earth's permanent magnetic field which is due to magnetised matter, it may have some share in producing the secular variations.

15. The self-energy integral of the component $P_1(\cos\theta)$ of the disturbance field may next be noticed. It is found to be

$$\tfrac{1}{2}(\delta H)^2b^3 \quad . \quad . \quad . \quad . \quad (18)$$

Since a and b are nearly equal, while δH is always small compared with H_0, the value of (18) is always much less than that of (17). To take a numerical case, a storm for which the maximum value (δH_0) of δH is 100 γ (10^{-3} c.g.s.) may be considered; this, which will be regarded as the "standard" storm, is of fairly considerable magnitude. The joint integral (17) is equal to

$$0{\cdot}316 \,.\, 10^{-3} \,.\, (6{\cdot}4 \,.\, 10^8)^3 = 8{\cdot}3 \,.\, 10^{22} \text{ ergs.} \quad . \quad (19)$$

while the self-integral (18) is

$$\tfrac{1}{2} \,.\, (10^{-3})^2 \,.\, (6{\cdot}5 \,.\, 10^8)^3 = 1{\cdot}4 \,.\, 10^{20} \text{ ergs} \quad . \quad . \quad (20)$$

or about $\frac{1}{600}$ of the former.

16. These estimates need to be revised on account of the

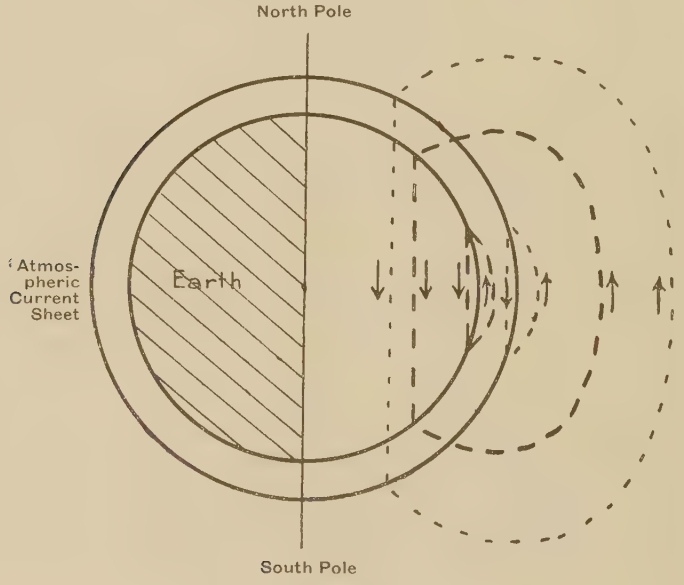


FIG. 2.

induced earth currents and their associated magnetic field. The additional terms in the disturbance potential will be

$$\begin{cases} -\delta_i H \,.\, a^3 \cos\theta / r^2 & (r > a) \,. \quad . \quad . \quad (21) \\ \delta_i H \,.\, r \cos\theta & (r < a) \,. \quad . \quad . \quad (22) \end{cases}$$

where the latter is assumed in accordance with § 8; $\delta_i H$, which is positive, denotes the part of the diminution of the horizontal force at the equator which is due to the internal currents. In the former equations (11) to (18) δH should be replaced by $\delta_0 H$, denot-

ing the part of this diminution which is due to external currents. The whole diminution δH is then given by the equation

$$\delta H = \delta_0 H + \delta_i H \quad . \quad . \quad . \quad . \quad (23)$$

In the calculation of the joint-energy integral $\delta\Omega$ occurs linearly, so that the parts arising from $\delta_0 H$ and $\delta_i H$ can be considered separately. The latter is readily found to be

$$- H_0 \cdot \delta_i H \cdot a^3 \quad . \quad . \quad . \quad . \quad (24)$$

Hence, recalling the value of (17), with $\delta_0 H$ substituted for δH, the whole joint-energy integral is found in this case to be

$$H_0(\delta_0 H - \delta_i H)a^3 \quad . \quad . \quad . \quad . \quad (25)$$

The self-energy integral, similarly, is easily verified to be

$$\tfrac{1}{2}(\delta_0 H - \delta_i H)^2 a^3 + \tfrac{1}{2}(\delta_0 H)^2(b^3 - a^3) \quad . \quad . \quad (26)$$

17. The expressions (25), (26), on comparison with (17), (18), well illustrate how the induced currents tend to minimise the changes in the magnetic field, and the energy corresponding to a given surface change of magnetic force. The vertical intensity, in particular, is much diminished. The internal currents are greater, and the excess energy consequently less, the more rapid the variations of the external currents. Hence, proportionately to the corresponding outer currents they will be larger in intense than in moderate storms, on account of the more rapid progress through the characteristic phases in the former case (§ 3, footnote); and in a given storm they will be proportionately much larger for the harmonic components corresponding to the local, rapidly changing, irregular magnetic variations * than for the world-wide change (going through its phases once only throughout the period of the storm) associated with the above harmonic $P_1(\cos\theta)$.

18. In revising the former numerical estimates of the energy associated with the standard storm for which δH_0 is 100 γ, it will be assumed that one-quarter of this variation is of internal, and

* Not only will the earth currents be greater (in proportion) for the latter, but they will be much more concentrated near the surface, in accordance with a well-known property of conductors in varying magnetic fields. This explains an observation made long since by Airy from the Greenwich earth-current records, viz. that while the earth currents showed variations corresponding to magnetic *disturbances*, they seemed to have no corresponding relation with the diurnal magnetic variations, even when these were of analogous magnitude, as in summer. The outer "moist" stratum of the earth's crust is an electrical conductor, but below that, down to a depth of about 200 kms. (as I have shown elsewhere) the strata are relatively poorly conducting. The "diurnal" earth currents flow mainly in the inner conducting core beneath this depth, but during rapidly varying magnetic disturbances the earth currents in the outer crust rise to importance and partly shield the interior. The main world-wide changes of magnetic force during a storm are sufficiently slow to be classed with the diurnal variations as regards the rate of change of the magnetic field, and therefore also as regards the seat of the corresponding earth currents.

three-quarters of external origin. This division between the two sources is in general accordance with that which, in a study of the diurnal magnetic variations, I have found to hold good in their case. Hence

$$\delta_0 H_0 = 0{\cdot}75 \,.\, 10^{-3} \text{ c.g.s.} \qquad \delta_i H_0 = 0{\cdot}25 \,.\, 10^{-3} \text{ c.g.s.} \quad (27)$$

The former value (19) of the joint-energy integral is now halved, *i.e.* its value is

$$4{\cdot}2 \,.\, 10^{22} \text{ ergs} \quad . \quad . \quad . \quad . \quad (28)$$

while the self-energy integral (20) is much more reduced, to

$$0{\cdot}4 \,.\, 10^{20} \text{ ergs} \quad . \quad . \quad . \quad . \quad (29)$$

The maximum excess energy E_0 of the magnetic field during the storm consists of the sum of (28), (29), and of the self-energy integrals of all the other harmonic components of the disturbance potential. The sum of all these may, however, be neglected, together with (29), as being negligible in comparison with the joint-energy integral (28) for the main component $P_1(\cos\theta)$. The remaining harmonics are of higher degree and order, and their influence falls off much more rapidly, with increasing distance from the current sheets, than in the case of $P_1(\cos\theta)$—on account of the higher degree of the factors r^n or r^{-n-1} in the expressions of types (1) and (2). Also the internal currents are much more efficient in their case in reducing the associated energy of the field.

19. It may be concluded, therefore, that during a magnetic storm the excess energy E of the magnetic field steadily increases during the forced phase to a maximum value E_0, which is approximately equal to

$$H_0(\delta_0 H_0 - \delta_i H_0)a^3 \quad . \quad . \quad . \quad . \quad (30)$$

Neglecting the slight variation which may occur in the ratio $\delta_i H_0/\delta_0 H_0$ for storms of different intensities, and writing

$$\delta_0 H_0 = \tfrac{3}{4}\delta H_0 \qquad \delta_i H_0 = \tfrac{1}{4}\delta H_0 \quad . \quad . \quad (31)$$

where δH_0 is the maximum mean equatorial diminution of horizontal force, the expression (30) may be written in the simple form

$$\tfrac{1}{2} H_0 \delta H_0 \,.\, a^3 \quad . \quad . \quad . \quad . \quad (32)$$

Finally, taking into account the loss of energy in overcoming electrical resistance to the currents in atmosphere and earth during the forced period of the storm (§ 5), the whole energy of the storm may be estimated as being approximately $2E_0$, or

$$H_0 \delta H_0 \,.\, a^3 \quad . \quad . \quad . \quad . \quad (33)$$

As in (19), the value of this for the standard storm is found to be $8{\cdot}3 \,.\, 10^{22}$ ergs, or, in round figures, 10^{23} ergs. It may be noted

that the forced phase in a storm of this magnitude would normally last for about fifteen hours.*

20. This energy is supplied through the medium of the electromotive forces which generate the currents in the atmosphere current sheet. It is ultimately degraded into heat energy in the atmosphere and earth, and although this process is only half accomplished during the forced phase of the storm, the bulk of the 10^{23} ergs is supplied during this period of fifteen hours. Hence the rate of supply of energy during the second phase of the standard storms is of the order of 2.10^{18} ergs per second.

21. It is of interest to translate these estimates into more familiar terms. The rate of supply of energy is equivalent to 270 million horse-power (1 H.P. $= 7{\cdot}46 . 10^9$ ergs per second) or, in calories (1 calorie = 1 gramme–degree–centigrade $= 4{\cdot}2 . 10^7$ ergs), $4{\cdot}8 . 10^{10}$ per second. The direct solar radiation on one square centimetre of the earth's surface supplies 1·95 calories per minute, and the total quantity which would be received on one hemisphere in projection is therefore $4{\cdot}2 . 10^{16}$ per second, or approximately 2.10^{24} ergs per second—nearly one million times as great as that involved in the magnetic storm. Thus, while the expenditure of energy during a magnetic storm is very great, it is quite insignificant compared with the supply continually being received by the earth through the ordinary solar radiation.

22. The above results may be compared with Lord Kelvin's well-known estimate of the energy involved in a magnetic storm. He considered a brief interlude of fifty minutes' duration, in the remarkable storm of June 25, 1885, in which there occurred a diminution of 100 γ in the horizontal force at several stations (though much less at others). On the hypothesis that this was merely the local indication of a change in the whole magnetic field of the sun, regarded as a simple magnet, he calculated † that "to produce such changes by any possible dynamic action within the sun, or in his atmosphere, the agent must have worked at something like 160 million million million million horse-power (12.10^{35} ergs per second), which is about 364 times the total horse-power ($3{\cdot}3 . 10^{33}$ ergs per second) of the solar radiation." ‡ This result was interpreted as being "absolutely conclusive against the supposition that terrestrial magnetic storms are due to magnetic

* *Cf.* fig. 6 (middle diagram, Bombay) of the paper cited in § 1, footnote. In the present paper it is assumed, for the sake of simplicity, that the minimum epoch of a storm is arrived at simultaneously all over the earth. The fact that this is not quite in accordance with observation (minimum epoch seeming to occur later in high than in low latitudes) implies that besides the harmonic $P_1(\cos\theta)$, the world-wide magnetic variation averaged round each circle of latitude involves also a term $P_3(\cos\theta)$. This, however, would scarcely affect the joint-energy integral, which yields the leading term in the expression for E_0.

† Lord Kelvin, *Proc. Roy. Soc.*, 52, 307, 1892. The details of the calculation were not given, but have been supplied by Prof. Schuster in his paper on "Sun-spots and Magnetic Storms." *M.N.R.A.S.*, 65, 187, 1905.

‡ Here the radiation in all directions is included, and not that only which falls upon the earth, as in § 21.

action of the sun, or to any kind of dynamical action taking place within the sun, or in connection with hurricanes in his atmosphere, or anywhere near the sun outside." He even went so far as to add that "it seems as if we may also be forced to conclude that the supposed connection between magnetic storms and sun-spots is unreal, and that the seeming agreement between the periods has been a mere coincidence."

While these negative statements are in excess of what can be strictly asserted, this striking calculation at least excludes one hypothesis which in the first instance it was proper to test.

The above estimate of the energy made by Lord Kelvin is clearly altogether too great. The other calculations which have been made have usually much underestimated the energy. The necessary distinction was not drawn between the world-wide and the local changes in the magnetic field, and the importance of the joint-energy integral was missed. Only the "self-energy" of the disturbance was considered at all, and one result of this has been that the expressions arrived at depended on the square of the disturbing magnetic force.

23. The total energy of magnetic disturbance in the course of a period of years can be roughly calculated, if the average value of the sum of the associated reductions of horizontal force (δH_0) can be estimated, taking into account all the magnetic storms which occur. The admirable study of magnetic disturbance at Bombay, made by Dr N. A. F. Moos,* is of much assistance in this calculation. Dr Moos has tabulated the principal disturbances during the thirty-three years 1872–1904, including all those in which the local value of the horizontal force changed through 70 γ or more; the number found was 347. For the 134 greatest of these the average variation of horizontal force throughout the first day of the storms was determined (*cf.* fig. 1), and the mean local value of δH at minimum epoch was found to be approximately 100 γ. At the equator this would be slightly greater. Hence it would appear that on an average (here taken over three sun-spot cycles) there occur annually about *four* storms of our "standard" magnitude. In addition there probably occur twice as many storms for which the approximate value of δH_0 is 50 γ, and there are many still smaller disturbances. Probably the average annual sum of the values of δH_0 for all the extensive magnetic disturbances is about 1000 γ, corresponding to ten standard storms. Hence the average yearly total energy of terrestrial magnetic disturbance may be estimated in round figures as being 10^{24} ergs.

24. Besides this energy supplied through the medium of the electromotive forces in the external current sheet, there is other energy associated with a magnetic storm, such as that involved in auroral phenomena. Probably the terrestrial part of the latter energy (which we may define as the part which remains in the earth, finally as heat) is insignificant compared with the energy

* N. A. F. Moos, *Colaba Magnetic Observations*, 1846–1905, 2, ch. 10.

of the storm as here calculated. There may be, however, and in my view there are, quantities of energy concerned in the mechanism which produces the atmospheric electromotive forces, which are much more extensive than those quantities which by this means are transformed into the energy of the storm, and which remain in the earth thereafter in the form of heat.

The view held with regard to such possibilities will depend on the ideas which are formed as to the source and mode of transformation of the energy of the storm. The above calculation of the energy is not based on any particular hypothesis as to this matter, but the question may appropriately be considered at the present stage in the discussion.

25. The tendency in the past has been to discredit hypotheses which attribute the production of magnetic storms to streams of electric corpuscles projected from the sun, at any rate in so far as these hypotheses regard the sun as the source of the energy involved. The connection between storms and the sun is well attested, but the sun's action has been regarded as akin to that of the trigger of a gun, *i.e.* as releasing and permitting the transformation of energy from some terrestrial store.

Two particular possible sources of the energy which have been mentioned are the heat energy of the atmosphere and the rotational energy of the earth. The former has been rejected because of the absence of any indication as to how the transformation to electromagnetic energy is, or with probability could be, brought about. The hypothesis would involve the semi-spontaneous execution of a cycle of energy transformation within the atmosphere, from heat energy through electromagnetic energy back to heat energy again. Moreover, only a small part of the atmosphere can be concerned in a magnetic storm, and in this the changes of temperature would need to be quite improbably great.

The other suggested source, the kinetic energy of the earth's rotation, comprises approximately $2{\cdot}6 \cdot 10^{36}$ ergs. This represents a very considerable store, so great that the withdrawal from it of the energy of magnetic storms, at the estimated average rate of 10^{24} ergs per annum (§ 23), would produce a secular retardation of only about 1″ per century per century. The secular acceleration of the moon seems to indicate that there is actually a secular retardation of the earth's rotation of about 10′ per century per century, so that the retarding effect of magnetic storms, on the present hypothesis, is almost negligible compared with that of other forces which appear to be acting. The energy involved in magnetic storms is of the same order as that dissipated in oceanic tidal friction, as estimated by Jeffreys (1″·2 seconds per century per century).*

* H. Jeffreys, *Nature*, 99, 405, 1917; *cf.* also *ibid.*, 100, 145, 187, and R. O. Street, *Proc. Roy. Soc.*, A, 93, 348, 1917. The estimate was based on considerations of ordinary molecular viscosity. Major G. I. Taylor has shown that eddy viscosity is of much greater importance, especially in nearly land-locked seas.

The suggestion that the kinetic energy of the earth's rotation is actually the source of the energy of magnetic storms clearly cannot be rejected on the ground of any insufficiency in the store. But it is partly open to the objection which led to the rejection of the previous suggestion which has been mentioned—no satisfactory explanation seems to be available to account for the transformation of the energy into that communicated to the earth's magnetic field.

26. My own view is that the sun is the source of the energy, and that this is transmitted by streams of electric corpuscles, which, however, do not produce the storm by direct magnetic action. The theory I have recently outlined was arrived at by considering the nature of the atmospheric motions by which the production of the necessary electromotive forces in the upper atmosphere was to be explained, and without reference to energy considerations in the first place. On this view the corpuscles which enter the atmosphere, after being deflected in the earth's permanent magnetic field, both ionise and charge the "absorbing" layer. The air being rendered electrically conducting, the charge will tend to distribute itself uniformly over the spherical layer; the estimates of the conductivity which I have made suggest that the conductivity is sufficient to allow the re-distribution to take place within a small fraction of a second. Hence the electrostatic field within the spherical electrified layer, and in particular near the earth's surface, will remain unaffected by the electricity injected during a magnetic storm. This agrees with a fact of observation which has generally seemed irreconcilable with corpuscular theories of magnetic storms. But while the charge rapidly becomes uniformly distributed, the inequalities of conductivity produced by the varying intensity of corpuscular injection over the earth may endure for hours.

27. The production of the electromotive forces which are the medium of transmission of the energy of a magnetic storm is attributed on this theory to the upward movement of the outer atmosphere under the influence of the mutual repulsion of the imprisoned electric charge. The accumulation of charge continues until the electrostatic repulsion overcomes gravity, when the electrified gas is impelled upwards; the atmosphere thus proceeds to lose both its charge and part of its own substance. I have shown that this vertical movement of air produces, by induction across the horizontal component of the earth's magnetic force, electromotive forces of the type associated with the forced phase of a storm.

If this theory be correct, the immediate source of the energy of the storm is clearly the electrostatic energy of the charged layer; and this electrostatic energy is a transformation of that part of the kinetic energy of the corpuscles which is used up in charging the earth's atmosphere against the electrostatic repulsion of the charge accumulated up to the given time. The energy of the magnetic storm is consequently a fraction of that which is

given up by the sun when it projects the stream of corpuscles from its own surface.

28. In his valuable criticism of stream theories of magnetic storms, Dr Schuster has emphasised the difficulties under which these labour, owing to the enormous energy expenditure required from the sun in order to project the corpuscles from its surface.* The theories of stream action with which he was dealing were, however, of the type in which the terrestrial magnetic disturbances are attributed to the direct magnetic action of the streams. But the speed and density of charge required in such streams are altogether greater than are necessary in the streams contemplated by the present theory, so that the above difficulties do not arise. The exposition of these and further points in connection with the theory will be undertaken in a subsequent paper.

Micrometrical Measures of 480 *Double Stars.* By W. S. Franks.

The following list of double-star measures is the sixth of the series, its predecessors having been published from time to time in the *Monthly Notices*. The instrument used throughout was the $6\frac{1}{8}$-inch Cooke equatorial of the Brockhurst Observatory, with powers of 235 and 345 on the micrometer, the value of one revolution of the screw being $25''{\cdot}38$. In all cases *bright-field* illumination was used. With so small an aperture as this, it was realised from the first that it was unwise to enter into competition with the many powerful telescopes now in use in the measuring of close double stars. It was therefore decided to attempt only what was well within the reach of the instrument; so the measuring of the wider doubles (for a long time systematically neglected) became the main feature of the work. To begin with, the whole of the OΣ App. stars (*Poulkova Catalogue*, part ii.) and Σ's two Appendices were measured, as also many of the wider doubles of OΣ. Afterwards the *Mensuræ Micrometricæ* (Lewis's revision, *Memoirs R.A.S.*, 56) furnished most of the material for the working lists, with occasional selections from Herschel, and other miscellaneous objects. As the magnitudes of the Herschel stars are often very loosely given, I have usually substituted my own estimates, according to Struve's scale. Many of *h*'s pairs are too faint to measure satisfactorily with a 6-inch O.G., and require a larger aperture to deal with them. Up to the present time, measures have been made of some 1850 double stars, of which about 1360 are Σ and OΣ objects (including *rej.* pairs) and the remaining 490 miscellaneous doubles.

For other additions to the working list I am indebted to the Rev. T. E. Espin, who has kindly placed much material at my service, including a list of the little-known Argelander double

* A. Schuster, *Proc. Roy. Soc.*, A, 85, 44, 1911.

stars (*Bonn Obs.*, 6.), published in the *Journal of the R.A.S. of Canada*, Nov.–Dec. 1908. [In the measures Arg, followed by a number in parentheses, refers to this list.] And I have also had valuable assistance from Mr. F. A. Bellamy, who has helped me considerably with copious notes of objects from Burnham's *General Catalogue*, the index volume being kindly lent to me by Professor Turner.

Abbreviations.—Arg, *Argelander*; A.G., *Astronomische Gesellschaft*; β, *Burnham*; Cin, *Cincinnati Obs.*; De, *Dembowski*; Du, *Dunér*; Es, *Espin*; Fr, *Franks*; Gla, *Glasenapp*; Gr, *Groombridge*; *h*, *Sir J. Herschel*; Hh, *Sir J. Herschel's Synopsis of* H's *double stars*; Hd, *Harvard*; Ja, *Jacob*; Kr, *Krüger*; Kü, *Küstner*; Lac, *Lacaille*; Ll, *Lalande*; O.S, *O. Stone*; OΣ, *O. Struve*; Schj, *Schjellerup*; S, *South*; Sh, *Herschel and South*; Σ, *W. Struve*; W, *Weisse*; Wn, *Winnecke*. Ang, *angle*; D, *distance*; <, *increasing*; >, *decreasing*; s, *slow or small*; p.m., *proper motion*; c.p.m., *common proper motion*; cols, *colours*; orge, *orange*; yel, *yellow*; ylsh, *yellowish*; grn, *green*; grnsh, *greenish*; bl, *blue*; blsh, *bluish*; prp, *purple*; prpsh, *purplish*; wh, *white*; p, *pale*; o, *no colour seen*; v, *variable*; Cl, *cluster*; "Cel. Obj.," Webb's *Celestial Objects*; Quad, *quadruple*; Trip, *triple*.

Measures of Double Stars.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
Σ 1	0 3·7	+36 40	8·5, 10·0	286·3	9·50	1917·731	2	
Es*	5·1	49 19	7·8, 7·8	293·5	77·60	·843	2	BD+49° 17; yel, ye
Es*	5·9	48 59	8·0, 8·2	310·6	66·72	·847	2	BD+48° 30; wh, wl
De 1	7·4	45 51	9·0, 9·3	72·9	13·07	·781	2	Ang>s?
Σ 16	11·4	54 6	7·7, 9·0	38·5	5·47	·731	2	wh, p. bl.
Es*	12·4	53 9	7·2, 9·8	65·5	14·38	·847	2	BD+52° 38; p. y blsh.
h 3	12·7	12 29	9·0, 9·0	82·1	11·41	·932	2	
Gr 34	12·7	43 27	8·3, 10·0	56·5	39·37	·894	4	BD+43° 44. C.p.m
Es 42	17·4	+53 46	8·4, 9·6	191·7	10·35	·806	3	=Es 154.
Schj 1	21·7	− 6 5	8·7, 9·5	27·7	25·03	·806	2	ylsh wh, o.
OΣ 10 *rej.*	22·3	+15 28	6·3, 9·2	237·7	103·44	·932	2	D< ; p.m. ; wh, bl.
Es 2	25·6	56 14	8·8, 9·4	111·0	6·44	·932	2	wh, wh.
S 386	26·6	27 58	8·6, 8·6	195·9	42·40	·784	2	wh, wh.
h 1032	26·7	28 58	8·3, 9·0	246·1	12·22	·847	2	wh, blsh.
Es 3	29·4	56 2	8·5, 9·4	156·9	8·52	·932	2	wh, blsh.
A.G. 6	30·3	11 17	8·2, 10·0	12·1	61·76	·793	2	Ang and D<; yel, o.
Hd 28	30·5	+ 2 46	8·8, 9·8	341·2	22·85	·940	2	Probably identical w Hd 29.

* Unnumbered.

Measures of Double Stars—continued.

n.	R.A. 1900. h m	Decl. 1900. ° ′	Magnitudes.	Pos. Ang. °	Distance. ″	Epoch.	Nts.	Remarks.
2	0 30·7	+29 27	7·9, 8·7	26·7	5·59	1917·743	2	AB. Ang> ; wh, wh.
9	35·7	− 7 47	6·5, 10·0	323·6	6·31	·752	2	yel, o ; Ang and D<.
12	39·0	+74 26	5·6, 10·0	160·1	36·28	·685	2	21 Cass. ; wh, o.
2	40·1	54 26	8·4, 9·3	117·0	12·91	·932	2	D> ? BD+54° 144.
46	40·2	49 18	8·5, 9·5	255·9	10·24	·855	2	ylsh wh, o.
7	47·1	60 34 Fr	5·2, 8·7	170·6	128·68 AB	·847	2	B.A.C. 239 ; p. yel, prpsh ; p.m.
1	48·2	4 28	8·5, 9·8	337·1	8·72	·752	2	Ang>s ?
9	49·5	83 8	8·5, 9·7	18·3	24·01	·743	2	Ang and D<.
3	51·2	59 49	8·4, 8·6	201·2	20·41	·806	2	BD+59° 150, 149. Ang and D> ?
rej	1 1·6	79 48 Fr	9·0, 9·3	342·4	15·75	·943	2	Qu. Cin Ang 14° 8 ?
28	9·0	73 29 ,,	8·0, 8·2	203·7	61·45	·943	2	C.p.m. ? p. yel, p. yel.
4	11·1	37 58	8·0, 10·0	322·7	13·70	·763	2	
7	12·1	20 33	8·3, 10·0	67·9	20·81	·752	2	
30	13·8	57 42	5·2, 7·5	229·8	133·87	·752	2	ϕ Cass. ; p. yel, lilac.
(Fr)	18·0	58 26	8·0, 9·0	4·7	49·84	·836	2	BD+58° 230, 231 ; yel, bl.
	18·4	52 40	8·6, 8·9	339·6	28·34	·852	2	BD+52° 333 ; yel, wh, wh.
21	20·9	63 58	8·7, 9·7	274·0	13·03	·775	2	Ang and D>.
22	21·7	3 1	7·9, 9·0	326·6	5·97	·817	2	wh, bl ; Ang>s.
8	23·3	82 50	8·5, 9·0	80·0	12·82	·770	2	Ang and D<.
	26·3	54 14 Fr	9·0, 9·0	195·3	16·30	·852	2	BD+53° 326.
	26·8	+53 46 ,,	9·0, 9·2	27·9	15·41	·852	2	BD+53° 327.
52	26·8	+19 32 ,,	7·0, 7·3	117·0	78·47	·820	2	ylsh wh, ylsh wh.
0	27·1	+69 24	8·0, 9·0	187·8	7·43	·770	2	
e 5	28·9	−12 19 Fr	8·7, 9·0	29·8	13·59	·835	2	Ang>D< ?
7	40·8	+38 26	8·5, 9·0	115·9	12·64	·817	2	
91	41·7	61 20	8·5, 8·9	152·1	27·57	·793	2	wh, wh.
4	43·3	33 34	8·7, 9·0	95·8	9·63	·817	2	
(5)= G. 24	45·5	45 19	8·6, 8·8	170·9	15·85	·793	2	BD+45° 464 ; wh, wh.
6 *rej.*	45·6	28 11 Fr	8·5, 9·9	324·1	24·65	·948	2	wh, o.
94	50·3	68 53 ,,	8·8, 9·6	268·6	25·98	·943	2	
	50·7	29 58	8·2, 9·2	306·2	53·39	·852	2	BD+29° 330.
	51·3	30 32	7·6, 9·2	270·1	66·62	·852	2	BD+30° 303.
1	54·3	73 22	6·2, 8·5	191·8	5·56	·817	2	wh, bl.
2	55·0	+76 48 Fr	5·5, 9·5	198·0	94·82	·811	2	47 Cass. ; p. yel, prpsh.

* Unnumbered.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
Σ 198 *rej.*	1 55·0	+ 6 13	8·5, 9·0	116·9	45·27	1917·943	2	wh, wh.
Σ 221	2 4·1	+19 53	7·7, 8·9	144·9	8·40	·883	2	p. yel, bl.
Hh 61	8·3	− 3 31	7·9, 9·1	135·5	81·44	·820	2	p. yel, blsh.
Σ 232	8·9	+29 56	7·5, 7·5	246·1	6·62	·883	2	wh, wh.
Σ 241	14·1	73 39	8·5, 10·0	283·7	20·40	·817	2	
Arg (7)	18·4	51 55	8·8, 9·1	243·8	17·76	·814	2	BD+51° 568; wh, wh
Σ 260	19·5	+53 50	8·2, 8·7	348·1	6·68	·865	2	
Σ 266	19·8	− 2 35	8·2, 8·7	267·6	7·47	·883	2	
Σ 267 *rej.*	21·4	+53 55	Fr 8·5, 8·5	24·3	92·79	·946	2	wh, wh.
h 2144	26·2	48 25	8·6, 9·5	259·2	26·48	·884	2	ylsh wh, o.
Σ 273	26·4	17 55	7·7, 8·7	358·4	6·84	·883	2	wh, wh.
Es*	29·7	34 15	5·5, 7·5	17·1	138·92	·854	2	15 Triang; orge, yl grn.
A.G. 44	34·8	34 24	9·0, 9·2	288·1	10·02	·873	2	
A.G. 46	35·0	39 50	9·0, 9·2	344·0	37·92	·824	2	BD+39° 603.
A.G. 54	44·3	11 44	8·9, 9·0	2·6	28·74	·948	2	Ang<D>.
A.G. 59	52·1	13 55	9·0, 9·6	85·9	24·65	·995	2	Ang<D>?
h 1129	56·0	69 50	9·0, 9·0	169·1	45·82	·946	2	wh, wh.
Es 558	3 0·0	45 22	7·5, 9·4	358·6	8·42	·964	2	wh, blsh.
Σ 350	0·9	20 15	8·0, 9·7	118·5	16·56	1918·010	2	
Arg (9)	17·9	43 55	Fr 8·8, 9·7	196·8	26·82	1917·964	2	BD+43° 705.
A.G. (Es)	18·6	46 40	,, 8·6, 9·4	315·2	10·52	·964	2	BD+46° 748.
Σ 406	25·5	4 48	7·0, 9·0	123·7	9·42	1918·010	2	wh, blsh.
Arg (10)	25·7	45 13	Fr 9·1, 9·8	164·5	27·84	·024	2	BD+45° 779.
h 2198	34·4	+54 17	,, 8·8, 8·8	308·2	41·92	·024	2	wh, wh.
h 2201	36·2	− 5 37	8·0, 9·5	36·3	41·62	·046	2	wh, o.
Σ 437	37·9	+31 48	9·0, 9·0	129·7	11·38	·037	2	
Σ 439	38·3	31 51	8·3, 9·2	38·7	23·35	·037	2	
Es*	38·3	39 44	Fr 7·2, 9·5	133·0	49·87	1917·995	2	BD + 39° 852; y wh, o.
Es 465	38·6	+50 7	8·7, 9·2	251·3	7·58	1918·037	2	
h 3588	40·0	−11 5	7·0, 9·0	229·1	47·24	·037	2	wh, prpsh.
Σ 443	40·2	+41 10	8·2, 8·8	52·7	8·40	·037	2	Ang<D>s.
h 1139	41·4	+70 11	Fr 8·0, 9·0	175·7	47·29	·048	2	p. yel, blsh.
h 669	48·1	+35 1	9·0, 9·2	261·5	11·95	·046	2	
Σ 468	48·1	− 2 8	8·5, 8·7	98·3	20·08	·037	2	
Σ 478	54·3	+11 16	8·2, 9·2	136·1	9·63	·046	2	
A.G. (Hd)	54·8	+54 10	8·8, 9·2	325·1	19·35	·072	2	

* Unnumbered.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
3615	3 58·5	−15 25	8·0, 9·0	157·0	25·24	1918·072	2	wh, wh.
G. (Hd)	4 7·8	+50 35	8·5, 8·9	284·5	45·23	·072	2	wh, wh; BD+50° 944.
573	10·6	30 33 Fr	8·0, 10·0	195·3	21·24	·072	2	ylsh, o.
g (Es)	14·3	59 23	6·0, 8·8	58·7	32·26	·072	2	wh, bl; BD+59° 793.
524	15·6	49 21	8·5, 9·5	56·1	6·82	·078	2	Lewis 49° 8′.
538	21·3	64 0	8·5, 9·7	219·5	7·50	·075	2	
546	21·2	18 53	7·7, 9·5	187·3	6·98	·078	2	ylsh wh, bl.
g (Es)	25·4	+52 36	6·7, 8·6	128·6	47·51	·075	2	yel, purp; BD+52°843.
653	26·0	−16 40	8·0, 9·0	156·3	42·53	·072	2	wh, wh.
124	27·3	− 3 23	5·9, 9·4	117·5	124·36	·075	2	wh, o; BD−3° 809.
563	29·7	+40 51	8·0, 9·7	31·5	11·81	·078	2	
56	33·6	58 33 Fr	8·7, 9·0	205·8	9·99	·077	2	
582	40·5	6 56 ,,	9·2, 9·2	149·5	25·26	·078	2	
598	42·8	+17 38	8·2, 9·7	318·9	9·53	·080	2	
57	48·0	− 1 26	8·3, 8·3	354·3	41·23	·078	2	wh, wh.
521	53·9	+39 3	9·0, 9·0	131·3	9·76	·080	2	
269	5 6·2	16 38 Fr	8·3, 10·3	60·1	20·30	083	2	wh, o.
r.	9·2	53 28 ,,	8·5, 9·7 (v)	58·7	47·09	·104	2	R Aur.; ylsh wh, ruddy.
569	11·6	45 9	7·8, 8·3	276·3	9·94	·127	2	wh, wh.
332	14·8	+33 17	8·3, 8·5	210·3	14·67	·083	2	wh, wh.
76	14·9	−18 37 Fr	6·3, 6·5	17·9	39·17	·083	2	BD−18° 1055–6; ylsh wh, ylsh wh.
(Fr)	21·3	+40 16	8·5, 8·6 AB ,, 8·7 AC ,, 8·6 AD	333·1 46·0 259·2	51·65 75·92 51·33	·234	2	Quad. BD+40° 1297, etc. Kite-shaped group.
83	21·4	+33 42 Fr	6·9, 8·7	49·0	97·43	·104	2	wh, bl; Ang>D<; p.m.
1759	21·7	−19 47 ,,	6·5, 8·5	319·1	26·97	·083	2	ylsh wh, bl; Ang< D>.
84	23·1	+33 22	8·0, 8·5	170·4	58·68	·127	2	wh, wh.
268	23·8	− 8 27 Fr	7·2, 9·7	300·2	27·07	·104	2	P. V 109; ylsh wh, blsh.
61	26·4	+ 2 45	8·0, 8·5	352·5	66·14	·127	2	ylsh wh, blsh wh; D>s?
45	29·9	− 6 4	8·5, 8·7	346·9	28·66	·127	2	Lewis −6° 14′.
40	30·2	+21 7	8·2, 9·0	118·9	21·70	·132	2	Lewis +21° 56′.
58	33·1	− 0·15	8·5, 9·0	296·7	11·25	·127	2	=Σ 765.
200	42·0	+56 53 Fr	6·4, 10·0	130·3	25·25	·131	2	29 Cam.; wh, o; Ang >D<.
3	44·7	− 4 30 ,,	8·5, 9·8	318·6	63·07	·131	2	ylsh, o.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
Σ 805	5 45·5	+28 25	7·7, 8·4	48·7	12·13	1918·132	2	wh, wh.
Σ 809	45·6	− 1 27	7·7, 8·8	96·3	25·26	·132	2	yel, lilac; Ang>.
Arg (14)= A.G. 101	45·6	+36 16	9·0, 9·6 *	32·1	10·24	·145	3	BD+36° 1276; Ang >rapidly.
Arg (15)	46·0	41 25	9·0, 9·5	114·5	22·87	·131	2	BD+41° 1289.
Arg (17)	51·6	10 13 Fr	8·6, 8·8	67·3	34·05	·132	2	BD+10° 939; wh, wh.
Σ 847	6 2·1	0 20	8·5, 8·8	261·3	24·84	·137	2	
Arg 13	3·0	57 3 Fr	7·8, 8·8	253·8	23·97	·137	2	Ang<D>; p.m. ylsh; BD+57° 945; wh, blsh wh.
Σ 884	13·0	47 10	8·5, 8·5	269·9	9·11	·137	2	
Σ 895 *rej.*	15·5	+ 5 47 Fr	8·5, 10·0	58·2	28·52	·134	2	p. orge, blue.
h 728	21·5	− 1 46	9·0, 10·0	259·8	25·44	·152	2	
h 392	26·9	+25 21 Fr	8·5, 8·5	37·3	23·90	·152	2	wh, wh.
Arg (Es)	31·6	40 26 ,,	7·5, 9·2	253·2	48·94 AB	·211	2	Ang<D>. BD+40° 1663; ylsh wh, 2 blsh wh.
,, Trip.	,,	,, ,, ,,	,, 9·0	90·1	61·54 AC	·211	2	Ang and D>; p.m. in A to the *s*.
Arg 15	31·9	−24 3 ,,	7·2, 8·3	234·3	41·02	·145	2	O. Arg S. 5344; wh, blsh.
S 529	31·9	+12 16	7·0, 9·0	151·2	70·29 AB †	·152	2	yel, ruddy
,, Trip.	,,	,, ,,	,, 8·0	168·5	162·96 AC	·152	2	Ang and D >in both pairs. ,, blsh wh
S 534	38·6	−22 21 Fr	6·3, 8·0	143·6	18·26	·145	2	p. yel, bl; BD − 22° 1505.
Arg (18)	43·0	+10 16 ,,	9·1, 9·3	105·2	13·05	·152	2	BD+10° 1277.
S 538	45·5	−24 1 ,,	7·5, 8·5	3·7	27·77	·155	2	wh, wh.
Arg (Es)	45·6	+51 46	8·0, 8·4	229·6	59·50	·160	2	BD+51° 1250; wh, wh.
h 740	46·6	0 35 Fr	8·4, 8·7	8·5	20·98	·160	2	In Cl. H VI 27.
h 2352	48·3	0 40 ,,	8·0, 8·7	23·1	21·56	·168	2	wh, wh; *h* 9, 10.
Σ 978	48·7	+38 2 ,,	7·0, 9·5	88·5	17·02	·168	2	yel, o; Ang>D<.
Σ 985 *rej.*	48·9	− 4 16 ,,	8·0, 8·7	322·1	32·85	·225	2	yelsh wh, blsh wh.
Σ 993 *rej.*	51·0	−11 44 ,,	8·0, 8·2	238·5	37·20	·168	2	wh, wh.
Σ 1002	55·8	+56 36	8·5, 9·0	317·1	30·29	·168	2	Lewis 8·2, 8·8.
Σ 1005 *rej.*	57·8	62 59 Fr	7·5, 10·0	90·0	33·40	·168	2	yel, o.
Σ 1012	58·6	28 17 ,,	8·5, 9·0	166·8	12·53	·168	2	Σ 8·2, 8·7.
Σ 1017	7 1·4	17 0	8·5, 9·2	255·2	12·44	·180	2	
Arg (Es)	3·3	50 4	7·6, 9·0	127·1	45·96	·180	2	BD+50° 1399; wh, bl.
Σ 1030	4·0	− 8 31	8·0, 9·2	43·3	15·77	·180	2	

* P.m. *h* 103°·3, 1832·72; Arg 78°·1, 1865·13; β 43°·9, 1902·78.
† A has p.m. of 0″·3 towards 194°·4.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
Σ 1036 *rej.*	h m s 7 5·3	° ′ − 6 7	9·0, 9·2	° 103·5	″ 15·91	1918·225	2	wh, wh.
Σ 3934	7·0	−21 38	8·0, 9·0	234·8	13·67	·155	2	
Σ 3938	9·6	−22 41	7·0, 8·5	249·0	19·89	·257	2	wh, blsh.
Σ 1052	9·8	−10 7	8·5, 8·7	23·4	20·03	·180	2	Ang<s.
Arg (20)	11·1	− 2 5	9·3, 9·4	330·8	15·10	·180	2	BD − 1° 1624.
Arg (Es)	14·2	+46 25	8·0, 8·2	265·3	80·65	·183	2	BD + 46° 1255.
Schj 5	20·3	− 5 32 Fr	8·5, 9·5	47·6	56·15	·180	2	
Σ 550	23·2	−18 17 ,,	7·0, 7·5	116·3	38·96	·257	2	ylsh wh, lilac.
Σ 1095	21·9	+ 8 57	8·3, 8·8	77·5	9·98	·180	2	
Σ 1096 *rej.*	24·3	50 22	8·0, 10·0	323·5	28·35	·228	2	wh, o.
Arg (21)	27·4	5 54 Fr	9·4, 9·5	136·5	21·76	·180	2	BD + 5° 1701.
Σ 555	32·1	−14 13	7·5, 7·7	228·5	95·33	·185	2	wh, wh } In Cl. H VIII
Σ 557	33·0	−14 13	7·4, 8·6	338·1	64·97	·185	2	wh, blsh } 38.
Schj 7	36·1	+ 9 26 Fr	8·5, 10·0	345·3	30·10	·185	2	
Σ 2418	44·8	20 16 ,,	9·2, 9·2	214·9	21·89	·183	2	
Es 590	47·8	43 25 ,,	9·1, 9·6	69·4	7·67	·257	2	
Σ 432	47·5	21 6	9·0, 9·0	270·7	11·92	·183	2	
Σ 1159 *rej.*	54·8	72 5 Fr	7·5, 9·5	93·3	35·50	·238	2	p. yel, bl.
Σ 1179	59·2	12 21	8·6, 8·6	204·2	20·45	·185	2	D<s.
Σ 1176	59·5	42 17	7·7, 9·3	28·4	22·24	·185	2	wh, o.
Σ 3308	8 3·7	+35 46	6·5, 10·0	265·9	46·10	·200	2	yel, o; BD + 35° 1767.
Σ 563	4·0	−19 34 Fr	7·0, 7·5	56·1	134·76	·185	2	ylsh, ylsh.
Arg (22)	8·5	+ 4 26	9·2, 9·3	327·0	51·88	·187	2	BD + 4° 1934.
Σ 1204	9·3	38 47	8·0, 9·0	102·7	11·62	·187	2	
Σ 1209 *rej.*	10·4	7 56 Fr	9·0, 9·5	152·4	20·29	·225	2	β's Ang 10° too small.
Σ 444	12·8	19 55 Fr	8·0, 9·5	96·8	35·16	·187	2	
Arg 18	15·8	64 28	9·0, 9·0	22·6	19·89	·225	2	wh, wh. BD + 64° 681.
Σ 1219	17·6	7 58	8·5, 8·5	260·8	11·40	·190	2	
Σ 3309	19·1	62 55 Fr	8·0, 9·0	129·8	25·25	·193	2	wh, bl. *h* 9, 9-10.
Σ 1220	19·4	24 41	8·0, 9·5	214·0	29·03	·190	2	Ang<D>s?
Σ 1222	19·8	37 53	8·0, 9·0	48·1	10·00	·190	2	
Es 18	20·2	+42 26	8·3, 9·2	236·4	12·70	·242	2	
Schj 10	20·6	− 0 5	7·5, 9·0	90·3	42·51	·189	2	wh, bl.
Σ 1231	23·6	+31 43	8·2, 8·7	210·9	24·70	·193	2	
Arg 20	24·2	−17 12 Fr	8·0, 8·3	173·0	15·23	·192	2	wh, wh. O. ArgS. 8579.
96	27·3	− 0 36 ,,	8·8, 9·0	20·9	39·86 AB	·259	2	} Triple.
,	,,	,, ,, ,,	9·1	139·7	59·63 AC	·259	2	
99	33·0	− 6 27	6·5, 7·7	202·4	61·04	·190	2	ylsh. wh, bl; Ang and D>; p.m.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
Σ 1256	8 36·0	+49 40	7·8, 9·3	214·0	26·21	1918·193	2	ylsh, blsh.
S 585	50·6	−17 51	Fr 6·0, 7·3	146·3	66·07	·187	2	very yel, blue; BD −17° 2691.
Σ 1294 *rej.*	51·1	+33 18	,, 8·5, 9·5	339·5	15·18	·193	2	
Σ 1299 *rej.*	54·8	+13 36	,, 8·5, 9·3	119·1	21·03	·192	2	
S 588	58·5	−17 16	,, 8·5, 8·5	327·5	30·79	·259	2	wh, wh.
Arg (Es)	58·6	+50 41	7·7, 8·1	258·0	79·06	·193	2	wh, wh. BD + 50° 1607.
h 116	59·1	− 2 29	Fr 9·0, 9·2	40·8	31·72	·192	2	
Σ 1304	9 6·2	+81 49	8·2, 9·0	318·6	23·91	·193	2	
W 21	10·0	− 8 21	Fr 7·7, 9·3	14·7	25·98	·197	2	ylsh wh, o.
h 808	11·5	+ 8 40	,, 8·5, 9·0	225·3	21·41	·197	2	
h 126	13·1	− 0 11	,, 8·5, 9·3	33·2	30·36	·197	2	*h* wrong quadrant; mags 9, 10.
Σ 1330 *rej.*	13·6	+67 36	,, 8·5, 10·2	302·8	24·96	·260	2	
Σ 1343	14·8	+ 5 26	,, 8·3, 8·5	271·7	9·87	·204	2	Σ 8·7, 92.
h 810	15·4	27 53	,, 9·2, 9·4	32·4	20·16	·197	2	
Σ 1359	25·8	56 41	8·5, 9·2	63·7	7·53	·204	2	Ang > s.
Σ 1366	28·8	53 45	7·8, 9·3	323·3	8·00	·204	2	ylsh wh, prpsh.
Σ 1368	29·1	+53 45	8·0, 9·5	220·5	21·56	·204	2	
Σ 1370	29·9	−12 9	8·5, 9·2	95·4	17·32	·204	2	
Es 428	30·7	+28 47	8·5, 9·2	200·4	12·63	·204	2	yel, bl
Σ 1376	38·7	43 41	8·1, 8·5	310·4	5·18	·204	2	Ang > s.
Σ 1391 *rej.*	49·4	51 40	Fr 8·5, 9·5	314·6	13·93	·242	2	
Hh 343	53·3	+11 27	,, 7·5, 10·0	355·9	50·69	·204	2	ylsh, o.
h 4303	10 15·6	−22 6	,, 7·7, 8·5	265·7	8·50	·205	2	yel, bl.
Σ 1430 *rej.*	19·8	+41 25	8·0, 9·5	235·2	22·84	·276	2	
Σ 1432	21·4	+30 11	8·0, 9·8	122·8	28·70	·207	2	
Σ 1437	26·1	74 21	7·2, 9·7	290·0	23·60	·211	2	ylsh wh, o.
Σ 1451	29·9	26 48	8·5, 9·5	268·9	8·28	·211	2	
h 487	30·5	30 39	Fr 9·2, 9·2	360·3	12·71	·205	2	
Σ 1452	30·7	3 4	,, 8·7, 8·8	328·8	10·12	·211	2	Σ 9·0, 9·1.
h 837	42·6	8 5	,, 8·5, 10·0	341·8	17·04	·205	2	ylsh, o.
Arg (Es)	47·4	44 31	8·5, 9·3	45·1	8·56	·214	2	BD + 44° 2024.
,,	49·2	+48 36	8·0, 8·7	158·6	46·80	·214	2	BD + 48° 1900.
S 617	48·4	− 1 43	6·6, 8·5	178·6	34·87	·214	2	yel, bl; BD − 1° 2459.
Σ 1492 *rej.*	52·1	+31 12	7·3, 10·3	164·3	21·48	·244	2	p. yel, o.
Σ 1496	53·1	+13 49	8·0, 10·0	352·6	19·43	·211	2	
h 1181	55·7	−17 47	Fr 8·5(v), 9·5	269·0	65·04	·214	2	R Crateris (Var); orge red. bl.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
26	10 55·9	+21 38	Fr 8·5, 9·0	174·8	16·56	1918·216	2	wh, wh.
2554	58·0	+44 52	,, 7·5, 9·0	279·6	49·05	·216	2	ylsh wh, bl.
3067	11 5·9	− 5 47	8·5, 9·2	234·7	21·01	·218	2	
n 3	13·4	+51 52	8·7, 9·6	206·8	6·78	·218	2	
1526	13·5	+ 3 22	8·8, 9·0	180·7	30·33	·218	2	
1530	14·7	− 6 21	7·8, 8·2	313·3	7·82	·218	2	wh, wh.
1531	15·4	+23 26	8·5, 9·5	165·7	23·16	·218	2	
6	24·6	−23 55	6·0, 8·7	77·9	8·63	·218	2	wh, bl; Lac 4757.
1549	27·4	+24 53	8·5, 9·5	115·4	13·77	·223	2	
508	37·0	40 14	Fr 9·0, 10·0	127·9	11·45	·220	2	
2583	38·5	14 5	,, 9·0, 9·0	56·6	30·20	·220	2	wh, wh.
1469	39·3	15 10	,, 9·0, 10·0	167·3	28·70	·220	2	
130	42·4	20 36	,, 8·0, 9·3	27·5	73·81	·223	2	Ang < D > s; curious miniature of 93 Leonis, 11′ to the north.
g (Es)	46·0	33 56	6·0, 8·5	273·4	45·38	·223	2	ylsh wh, bl. BD + 34° 2264.
511	47·0	19 25	Fr 7·5, 10·0	244·5	31·05	·220	2	wh, o.
3075	47·3	+ 8 7	8·8, 8·8	183·4	17·96	·223	2	Ang reversed, as Σ; and > s?
98	12 0·4	− 5 16	Fr 7·0, 10·3	270·8	80·16	·292	3	very yel, o.
g (Es)	3·3	+44 28	,, 8·1, 9·5	278·2	19·00	·253	2	BD + 44° 2162.
534	6·2	−16 13	,, 7·5, 9·0	285·5	6·46	·279	2	yel, prp; Ang < D >.
136	6·6	+82 17	6·0, 8·5	75·3	66·37	·253	2	yel, lilac; D < ; BD + 82° 356.
1616	9·3	9 21	7·5, 9·7	296·1	23·33	·287	2	wh, o.
1617 *rej.*	9·9	8 6	Fr 8·5, 10·0	261·3	31·58	·279	2	
1628	13·7	12 22	8·5, 8·7	240·3	9·96	·300	2	
n 4	17·4	58 39	Fr 9·0, 9·2	83·2	50·14	·253	2	Ang > D <.
1644	22·3	7 56	8·7, 9·2	244·8	20·27	·294	2	Ang and D > s.
g (Es)	23·3	48 27	Fr 9·0, 9·0	256·8	28·35	·279	2	BD + 48° 2023.
512	28·4	10 45	9·0, 9·3	265·3	21·81	·300	2	
1653	28·4	32 35	Fr 8·5, 8·8	342·8	7·80	·300	2	= Σ 1655. Mags var?
1658	30·0	+ 8 0	8·5, 8·9	260·3	110·64	·305	2	AC. Ang and D < ; p.m.
1665	33·5	− 4 46	8·5, 9·0	98·1	8·90	·300	2	
2618	36·0	+75 15	Fr 9·0, 9·2	203·8	30·28	·253	2	*h* 24°·6; Ang reversed.
1671 *rej.*	37·0	69 4	8·5, 10·0	201·6	29·28	·267	2	AB } Triple.
,,	,,	,, ,,	,, 9·8	145·7	40·00	·267	2	AC }
g (Es)	44·5	+42 47	7·4, 7·7	49·8	47·53	·287	2	BD + 43° 2261; wh, wh.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
Hh 406	12 48·4	+21 47	5·2, 9·3	124·8	28·57	1918·300	2	35 Comæ ; yel, blsh.
S 643	48·7	−17 29	8·0, 9·0	294·3	23·47	·279	2	wh, bl.
Σ 1697 *rej.*	51·5	+42 56	8·6, 9·2	217·7	28·39	·287	2	
Σ 1698	51·6	+75 11	8·2, 8·7	110·3	10·27	·310	2	
Σ 1705	55·8	14 55	8·2, 9·7	189·0	26·67	·310	2	
Σ 1713 *rej.*	58·7	26 20	Fr 8·5, 9·3	280·7	30·50	·310	2	Qu. error in β's distance
h 2640	13 2·6	12 50	9·3, 9·7	4·2	52·35	·334	2	
h 2649	14·3	54 52	Fr 8·5, 8·8	345·0	22·83	·316	2	wh, wh.
h 530	18·1	36 27	,, 8·5, 9·0	11·1	36·27	·334	2	ylsh wh, blsh.
Σ 1761	29·5	72 14	8·5, 9·0	72·3	20·26	·316	2	
h 2662	30·4	33 45	9·0, 9·3	284·7	25·00	·331	2	
h 228	30·6	10 43	7·0, 9·0	15·7	70·21	·334	2	yel, bl.
Σ 1773	36·6	8 6	9·2, 9·2	208·4	28·87	·316	2	AB } Triple.
,,	,,	,,	,, 9·7	102·2	57·02	·316	2	AC }
h 2673	37·2	60 15	Fr 8·7, 8·8	174·8	49·69	·316	2	*h* error of 100° in Ang
h 2676	39·0	50 32	7·8, 9·7	125·4	29·84	·316	2	wh, o.
Σ 1798	55·2	+78 54	7·5, 9·3	9·6	7·21	·316	2	yel, bl ; Ang>.
Howe 30	14 0·6	−12 36	8·2, 9·0	6·8	13·52	·334	2	
Σ 1833	17·3	− 7 19	7·0, 7·0	168·7	6·04	·334	2	wh, wh. P. XIV 62.
h 2721	21·2	+22 45	8·0, 9·0	138·5	100·71	·340	2	
Σ 1847	23·3	− 9 45	8·5, 9·8	261·1	25·90	·334	2	Ang and D<s.
h 1255	29·0	+41 53	Fr 8·5, 9·0	339·6	38·89	·340	2	wh, wh.
Σ 1861	31·9	12 36	8·7, 9·2	171·3	13·88	·340	2	Lewis 8·5, 9·7 ; Ang>
Σ 1862	33·1	15 19	8·5, 9·7	127·7	14·63	·346	2	
h 556	40·4	34 10	9·2, 9·5	334·2	32·19	·346	2	
Σ 1889 *rej.*	46·2	51 47	6·5, 9·8	87·7	15·71	·343	2	ylsh wh, o ; BD+5 1957.
A.G. 196	51·5	51 2	9·1, 9·5	139·3	24·95	·345	2	
Σ 1893	52·0	29 53	8·4, 10·0	235·8	19·52	·346	2	Ang>D>s.
S 666	57·6	75 18	6·5, 8·0	35·0	171·03	·349	2	p. yel, bl ; Ang a D>s.
Σ 1916	15 6·2	39 22	7·0, 9·5	330·4	9·61	·349	2	ylsh, blsh.
Σ 1924	9·8	26 7	8·5, 9·7	303·9	15·62	·349	2	Ang>s.
h 2773	13·3	41 47	8·9, 9·5	144·7	28·62	·350	2	
h 5492	13·5	14 33	9·0, 10·0	265·0	10·62	·351	2	*h* 245° ±.
Arg (Es)	18·8	45 29	8·6, 8·9	6·2	26·11	·350	2	BD+45° 2280.
Σ 1942	21·6	21 48	8·5, 9·5	90·3	9·20	·351	2	Ang>s.
Σ 1958	28·5	67 32	8·5, 8·8	339·2	30·02	·349	2	
Σ 1961	31·2	+43 51	8·7, 9·0	38·4	24·19	·351	2	Ang>D<.

Measures of Double Stars—continued.

yn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
788	15 31·3	+45 14	Fr 8·0, 8·0	304·4	58·12	1918·350	2	wh, wh.
966	36·5	−10 49	,, 8·0, 8·0	231·4	22·84	·349	2	wh, wh ; Σ 9, 9, vars ?
980	39·0	+81 23	8·5, 9·0	50·1	10·03	·349	2	Ang>s ?
. 199	49·3	+52 51	8·8, 8·9	254·8	9·61	·351	2	wh, wh.
213	53·4	−19 40	Fr 7·0, 7·2	317·4	17·24	·349	2	wh, wh ; Ang and D>.
002 *rej.*	54·	+83 35	8·0, 9·0	320·0	42·55	·349	2	ylsh wh, blsh wh.
. 201	16 0·8	49 14	9·0, 9·1	251·6	7·33	·354	2	wh, wh.
. 202	2·6	47 56	9·2, 9·3	283·9	22·95	·354	2	
29	3·7	+56 54	8·3, 8·8	140·4	25·87	·354	2	BD+57° 1636 ; wh, wh.
(29)	8·6	− 1 32	Fr 9·2, 9·3	242·0	20·68	·348	2	BD−1° 3150; blsh, blsh.
031 *rej.*	11·2	− 1 24	,, 7·5, 10·2	229·3	20·14	·349	2	ylsh wh, o.
on (β)	17·2	− 4 1	,, 7·5, 8·0	22·5	102·09	·349	2	yel, yel ; D<.
053 *rej.*	24·5	+31 21	8·9, 9·8	351·1	21·51	·350	2	
Zones	27·2	0 25	9·0, 9·1	305·3	7·11	·350	2	wh, wh.
54	40·0	44 5	Fr 8·4, 9·7	95·0	8·61	·354	2	Ang and D>.
(31)	40·2	47 46	,, 9·3, 9·4	85·7	20·18	·353	2	BD+47° 2373.
100 *rej.*	41·1	50 51	8·0, 10·0	295·0	23·86	·353	2	D<s?
(Es)	53·5	45 14	8·2, 8·3	252·1	48·97	·354	2	BD+45° 2470 ; ylsh wh, ylsh wh.
,,	54·1	+51 36	8·6, 8·8	283·0	45·61	·354	2	BD+51° 2156; wh, wh.
108 *rej.*	56·8	−11 45	Fr 8·0, 9·0	124·3	40·65	·349	2	wh, blsh ; D<.
j 14	17 13·5	+ 4 57	,, 8·3, 8·7	342·2	24·65	·368	2	ylsh wh, blsh wh.
86	14·1	+28 51	,, 8·0, 8·5	3·7	52·38	·368	2	wh, wh ; D>.
149	14·6	− 6 20	8·8, 8·8	24·1	7·25	·595	2	
249	19·9	+53 57	Fr 8·3, 8·7	75·7	62·54	·368	2	AB-C. Ang<?
. 209	24·3	+36 12	,, 8·8, 9·4	168·0	28·49	·368	2	D<.
169	24·5	− 8 20	,, 8·0, 10·0	95·3	14·86	·595	2	Ang<s.
964	29·2	−11 11	,, 6·0, 9·5	225·4	54·34	·368	2	wh, o ; BD−11° 4411.
(Es)	34·2	+43 12	,, 9·0, 9·1	153·5	9·39	·373	2	BD+43° 2774.
j 15	39·5	− 1 42	,, 7·5, 9·0	354·7	55·52	·368	2	wh, o.
. 213	42·1	+34 54	,, 9·0, 10·0	178·3	19·42	·373	2	
17	52·9	29 31	,, 9·0, 9·3	130·3	23·81	·373	2	wh, wh.
299 *rej.*	55·5	+84 5	8·7, 8·8	308·0	34·36	·373	2	wh, wh ; Ang>?
S 41	18 0·3	−18 55	Fr 8·5, 9·4	259·2	18·67	·598	3	Ang and D>.
92	0·8	−19 0	,, 7·5, 9·0	208·0	35·99	·600	2	Very wh, bl ; BD−18° 4789.
287 *rej.*	5·3	+ 2 34	9·1, 9·2	152·1	22·38	·401	2	Lewis 2° 24′.

* O. Stone 41 is 5′ *n* of *h* 592, and 0·5^{m} *p*.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
h 594	18 7·9	−18 50	7·5, 8·0	122·3	7·49 AB	1918·401	2	wh, wh } Triple.
,,	,,	−,,	,, 8·7	237·8	12·94 AC	·401	2	wh, wh }
Arg (34)= A.G. 217	9·0	+53 28	9·2, 9·5	244·8	15·93	·378	2	BD+53° 2043.
A.G. 220	14·3	51 0	9·1, 9·2	309·6	11·66	·378	2	
Σ 2326	15·3	81 28	7·7, 8·7	194·7	16·16	·376	2	wh, blsh ; Ang>.
A.G. (Es)	15·6	+29 52	9·0, 9·4	171·2	17·73	·378	2	
Σ 2306	16·5	−15 8 Fr	7·2, 7·5	219·9	11·13	·401	2	orge yel, v. bl, fine
Schj 17	21·7	+ 6 28 ,,	8·5, 9·5	350·6	50·26	·384	2	In Cl. ♅ VIII 72.
Σ 2338	27·6	38 35	8·5, 9·7	299·8	13·13	·379	2	
Schj 18	28·8	7 22	8·9, 9·0	197·3	46·08	·384	2	
Σ 2341	30·3	11 22	8·5, 9·7	265·6	15·42	·379	2	
A.G. 224	35·1	3 16	8·2, 9·2	348·0	22·05	·384	2	wh, blsh.
Σ 2377 *rej.*	37·8	63 26	7·0, 9·8	340·8	16·27	·376	2	ylsh, o.
Σ 2386 *rej.*	41·7	35 26 Fr	8·2, 8·7	20·8	20·81	·379	2	wh, wh.
Σ 2393	41·7	38 12	7·3, 10·0	22·5	14·44	·657	2	D< ; orge, bl.
h 1347	43·9	+28 19	8·0, 8·5	273·5	18·70 AB	·382	2	wh, wh } Triple.
,,	,,	,, ,,	,, 8·2	163·5	74·57 AC	·382	2	wh, wh }
h 5070	45·3	−22 8	8·3, 8·4	49·6	9·50	·449	2	Ang>s.
A.G. 226	46·2	+38 11	9·0, 9·1	51·4	26·13	·382	2	Ang>.
h 1352	46·2	29 41	7·2, 9·5	243·0	11·71	·382	2	wh, o.
h 870	46·9	10 11 Fr	8·9, 9·1	24·3	21·70	·401	2	
h 1354	49·5	+36 18	8·8, 8·9	5·7	10·16	·401	2	
h 2844	49·6	−17 46 Fr	8·5, 9·7	103·8	23·03	·594	2	Ang>.
h 1356	53·6	+45 19	8·6, 8·9	342·8	30·09	·381	2	
Schj 19	19 6·9	+ 0 45	8·5, 9·0	257·2	50·20	·453	2	
Schj 20	7·7	− 3 43	8·5, 9·0	229·9	60·64	·453	2	
S 715	12·0	−16 8 Fr	7·0, 7·5	14·8	8·21	·449	2	yel, bl ; D>s ; 16° 5253.
A.G. 228	12·4	+63 2	9·0, 9·1	102·6	37·54	·453	2	
A.G. 229	16·9	+ 9 0	8·2, 8·7	167·5	33·69	·461	2	D<rapidly.
h 2866	17·6	−18 12	8·0, 8·2	232·7	23·25 AB	·449	2	Forms a right-a triangle.
			,, 8·5	142·3	24·08 AC			Ang of AC<. T
Gla 8	19·2	−14 50	9·0, 9·5	65·5	24·36	·598	2	Ang>.
Schj 21	20·6	+ 4 38 Fr	8·2, 9·5	213·9	41·12	·471	2	
Σ 2517 *rej.*	20·6	22 34	8·7, 9·7	138·3	15·99	·473	2	
Σ 2521	22·1	19 42	6·0, 10·3	37·0	25·19	·655	2	P XIX 128 ; org blsh.
h 1418	30·5	+49 49	9·1, 9·6	11·2	29·03	·473	2	

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
2560 *rej.*	19 36·4	+23 29	7·2, 9·5	295·6	15·45	1918·477	2	wh, o ; BD+23° 3733.
G. 235	39·6	31 44	8·5, 9·5	167·3	11·04	·477	2	
rg (40)	42·7	46 8	Fr 8·5, 9·5	317·3	28·18	·477	2	BD+46° 2777 ; D>.
non Es	43·1	+ 9 47	,, 7·7, 8·7	294·2	49·86	·585	3	ruddy orge, ylsh wh. (*Cel. Obj.*).
2581 *rej.*	44·2	−11 39	8·0, 9·6	282·2	38·17	·473	2	ylsh wh, o.
G. 239	45·2	+51 39	Fr 8·4, 9·0	257·2	13·67	·477	2	ylsh, blsh.
G. 240	47·2	47 33	,, 8·7, 8·8	256·3	14·64	·485	2	wh, wh.
2593 *rej.*	48·6	+11 35	8·2, 9·7	235·0	12·67	·485	2	
hj 23	50·4	− 6 59	8·8, 9·4	23·8	35·80	·598	2	
G. 243	53·2	+30 48	8·5, 9·0	260·6	43·47	·488	2	wh, wh.
hj 24	54·9	11 37	Fr 9·0, 9·8	6·5	36·92	·666	2	
1458	55·4	10 54	8·8, 8·8	136·1	16·21	·497	2	wh, wh ; Ang<.
730	55·6	17 20	8·0, 8·7	15·3	114·09	·497	2	ylsh wh, blsh wh.
g 35	55·6	53 39	8·4, 9·0	225·3	7·55	·497	2	BD+53° 2332 ; Ang>.
G. 245	57·9	31 23	Fr 8·3, 9·5	357·1	12·52	·497	2	
2647	20 1·3	79 11	8·5, 9·5	261·6	8·44	·672	2	
506	4·7	37 50	8·6, 8·6	228·8	43·64	·502	2	ylsh wh, ylsh wh.
2638 *rej.*	5·3	+33 22	Fr 8·5, 9·5	73·5	16·51	·502	2	ylsh, o ; Ang >?
333	6·2	− 6 27	,, 8·4, 8·7	62·8	120·54 AB	·608	2	Ang >s.
87	6·5	+36 27	8·7, 9·3	298·7	9·61	·502	2	wh, wh ; D<s.
38760	8·8	− 1 10	7·0, 8·5	85·0	63·85	·508	2	wh, bl ; Ang<.
g 36	8·8	+57 0	8·7, 9·2	128·7	7·77	·508	2	BD + 56° 2368 ; ylsh wh, blsh.
2658	11·1	52 49	7·0, 9·1	113·2	5·55 AB	·672	2	Ang>D<in AC ; p.m.
			,, 10·2	207·7	47·18 AC			
G. 252	14·2	39 37	9·0, 9·2	127·1	10·63	·550	2	
G. 253	15·5	36 17	Fr 8·7, 9·1	117·7	9·64	·508	2	
*	16·1	51 53	8·3, 8·4	270·3	24·74	·505	2	BD+51° 2833.
r.	16·5	47 35	Fr 7·0 v, 8·0	51·3	63·52	·505	2	U Cygni ; orge red, blsh wh.
28	17·3	35 18	9·1, 9·8	257·5	24·55	·508	2	
g (45)	19·5	30 34	Fr 8·5, 9·0	178·2	20·79	·508	2	BD+30° 4010.
509	20·1	9 56	,, 8·8, 9·0	181·3	20·73	·655	2	*h* 9, 9–10.
yth	20·4	38 11	8·5, 8·8	295·5	65·74	·605	2	In Cl M 29.
682 *rej.*	21·4	25 1	8·2, 9·4	302·4	20·49	·511	2	ylsh wh, bl.
g (46)	22·1	41 35	9·0, 9·5	311·8	45·10	·511	2	BD+41° 3746 ; D>.
(Fr)	23·7	+53 42	8·3, 8·6	238·7	59·47	1917·870	2	BD + 53° 2406–4 ; yel, yel ; p.m.

* Unnumbered.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
S 750	20 25·6	+26 4	Fr 8·3, 8·3	322·2	67·26	1918·511	2	ylsh wh, ylsh wh.
Σ 2688	26·1	13 27	8·6, 9·8	173·7	6·99	·666	2	D<s.
A.G. (Es)	30·3	40 12	Fr 8·7, 9·0	267·1	9·36	·511	2	
h 1543	31·3	33 2	9·0, 9·2	26·5	16·72	·516	2	
Es 88	34·9	50 45	8·6, 9·0	127·4	7·66	·666	2	wh, wh.
Σ 2709	35·1	21 22	8·2, 10·0	305·7	9·36	·666	2	yel, o ; Ang>.
h 1565	38·9	22 38	Fr 8·2, 9·6	70·8	18·14	·516	2	
Arg 39	39·3	48 54	8·2, 8·4	121·3	9·66	·516	2	BD+48° 3193 ; Ang rapidly.
Es*	41·5	44 26	8·5, 8·7	80·0	53·60	·516	2	BD+44° 3569.
Es*	42·9	57 13	5·0, 8·7	161·1	66·34	·516	2	P. XX 332 ; p. yel, b Ang and D>.
Arg (47) =A.G.266	42·9	32 29	Fr 8·0, 9·0	85·4	11·62	·533	2	BD+32° 3936 ; D<
Es 134	43·5	63 10	8·5, 9·2	85·4	10·74	·533	2	Ang reversed.
Arg 40	47·8	44 56	8·8, 9·1	251·9	8·58	·533	2	BD+44° 3603.
Arg 41	48·5	+53 40	8·7, 8·8	182·7	9·53	·533	2	BD+53° 2506.
h 927	51·3	− 1 57	8·8, 8·8	347·7	31·20	·533	2	D<?
Hd Zones	52·4	+ 0 13	Fr 8·8, 9·3	139·5	41·27	·533	2	Ang<s.
h 3008	59·7	7 27	,, 9·0, 10·0	82·2	27·71	·666	2	
S 773	21 0·7	+35 7	9·0, 9·0	30·0	84·04	·550	2	
Σ 2755	2·4	− 0 34	Fr 6·7, 9·7	82·0	24·30	·631	2	orge, bl ; Σ 10·3 mag
S 777	3·3	+61 46	8·0, 8·5	299·2	7·13	·680	2	=Σ 2764.
h 1634	14·4	42 18	8·8, 9·0	137·6	26·93	·550	2	
h 281	16·4	+16 19	Fr 9·0, 9·5	332·4	14·72	·550	2	Ang>D<?
S 788	18·6	− 7 1	,, 7·0, 7·5	89·8	46·10	·550	2	ylsh wh, blsh wh ; A and D<.
Arg (Es)	18·6	+51 51	8·2, 8·4	40·3	28·96	·557	2	BD+51° 3051.
h 3027	18·8	70 45	9·0, 9·0	123·4	34·36	·557	2	
A.G. (Es)	30·8	42 55	8·5, 9·0	113·3	14·26	·557	2	
h 3039	30·8	0 14	Fr 8·8, 9·8	153·2	12·08	·631	2	
h 940	31·5	30 36	,, 9·4, 9·9	323·0	19·73	·631	2	
Kr 54	34·4	58 33	9·0, 9·2	128·7	16·74	·557	2	
Es 34	34·4	50 5	8·3, 8·7	70·2	39·73	·564	2	AC. wh, blsh.
A.G. 274	35·0	22 54	9·0, 9·5	153·3	8·64	·569	2	
h 3053	38·6	6 33	Fr 9·2, 9·7	194·5	26·62	·569	2	*h* 9, 10.
Arg (49)	45·1	42 13	9·2, 9·6	340·9	25·35	·569	2	BD+42° 4219.
h 5298	47·2	−16 17	8·5, 9·3	312·3	61·00	·569	2	
h 616	49·7	−12 27	Fr 8·0, 10·2	274·2	31·12	·631	2	

* Unnumbered.

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
800	h m 21 50·9	+62° 8′	Fr 6·7, 7·3	145°·5	62″·42	1918·564	2	p. yel, bl.
336	51·6	5 33	8·0, 8·5	223·7	99·94	·608	2	D>; Ang>s.
683	54·6	+44 13	Fr 8·7, 9·2	247·7	9·11	·569	2	
524	56·1	−16 6	,, 6·5, 10·0	291·4	101·77 AB	·613	2	p. yel, o } Triple.
,	,,	,, ,,	,, ,,	313·3	178 65 AC	·613	2	p. yel, bl } Triple.
951	57·1	+32 14	8·7, 9·0	78·5	9·95	·636	2	Ang>D<.
g (Es)	22 2·8	47 22	8·3, 8·7	278·8	38·63	·573	2	BD+47° 3717.
a 752	5·3	58 48	Fr 7·2, 7·3	316·2	21·76	·680	2	P. XXII 11, 12.
G. 280	5·9	31 11	,, 9·2, 9·7	180 6	10·91	·636	2	
742	6·3	67 14	8·5, 9·4	333·6	22·97	·573	2	Error of 10° in β's ang?
g (Es)	6·6	+47 25	7·0, 9·3	307·1	22·11	·573	2	BD+47° 3720.
2887	12·2	− 1 11	9·0, 9·0	29·8	8·64	·639	2	Ang<s?
38	19·3	+40 24	8·8, 9·0	52·0	6·73	·573	2	
08	20·3	−20 44	Fr 7·2, 8·2	149·8	7·30	·646	2	ylsh wh, blsh; D<s?
768	23·2	+47 18	,, 8·7, 8·8	8·4	23·87	·573	2	
on (Fr)	25·9	48 51	,, 6·5, 9·7	93·5	63·89	·591	2	BD+48° 3747; p. orge bl.
779	28·0	33 42	,, 7·8, 9·8	215·8	22·48	·573	2	ylsh wh, o; Ang>.
G. (Es)	28·4	52 36	8·5, 8·8	31·6	9·37	·573	2	wh, wh.
g 44	30·1	49 52	8·3, 8·4	168·8	8·02	·576	2	BD+49° 3897.
791	31·7	56 20	Fr 8·0, 9·3	60·8	17·26	·576	2	p. yel, blsh.
g (52)	32·0	45 31	9·2, 9·8	215·5	16·75	·575	2	BD+45° 3994.
G. 284	34·2	+36 46	9·0, 9·0	50·3	26·42	·576	2	
356	34·2	−28 51	6·3, 7·0	158·7	84·77	·649	2	AB. yel, bl.
106	34·9	+48 54	8·9, 9·5	266·1	8·37	·636	2	
G. 285	35·5	32 11	8·7, 8·8	312·5	38·48	·586	2	
941	41·1	+18 42	7·5, 10·2	262·5	11·37	·646	2	yel, o; Ang>D<; p.m.
944	42·7	− 4 45	7·0, 8·2	120·7	47·10 AC	·646	2	P. XXII 219; Ang>; p.m.
G. 288	44·1	+37 46	Fr 8·7, 9·3	179·4	18·29	·586	2	ylsh, blsh.
72	48·2	31 8	8·8, 10·0	194·2	24·04	·636	2	
47	49·8	19 48	Fr 8·4, 9·4	33·2	6·83	·586	2	Ang>.
20	53·1	72 18	,, 7·0, 8·0	279·4	120·75	·611	2	ylsh wh, blsh wh.
; (Es)	57·8	54 22	8·8, 9·0	252·2	30·58	·586	2	BD+54° 2899.
976	23 2·6	6 3	8·5, 8·8	195·5	17·62 AC	·652	2	Ang and D<; p.m.
79	3·3	21 34	8·5, 10·2	218·2	18·59	·676	2	
532	5·5	31 57	7·1, 8·7	76·8	57·80	·614	2	wh, blsh.
. 290	6·9	65 22	9·0, 9·1	265·3	14·94	·614	2	
176	7·9	+12 0	Fr 9·2, 9·2	342·8	27·98	·617	2	*h* 9, 9..

Measures of Double Stars—continued.

Syn.	R.A. 1900.	Decl. 1900.	Magnitudes.	Pos. Ang.	Distance.	Epoch.	Nts.	Remarks.
	h m	° ′		°	″			
h 3177	23 7·9	+10 8	Fr 8·5, 10·0	3·5	23·52	1918·617	2	*h* wrong quadrant.
Arg (Es)	14·4	48 22	7·3, 8·8	231·4	49·80	·614	2	BD+48° 3995; p. yel bl.
h 5397	18·5	−15 1	Fr 7·2, 8·5	337·8	65·72	·652	2	wh, wh; Ang and D<
Arg (Es)	33·2	+52 15	7·7, 9·0	142·4	19·64	·617	2	BD+52° 3487; yel, bl
Es 149	35·0	63 46	8·5, 8·7	118·7	6·28	·617	2	wh, wh.
h 991	36·6	21 54	Fr 9·1, 9·2	346·3	16·20	·622	2	*h* 9, 10.
A.G. 294	38·9	68 31	,, 8·5, 9·2	121·0	18·02	·622	2	Ang>D<.
h 3211	39·8	3 13	,, 9·1, 9·2	82·2	47·34	·622	2	Ang>?
S 835	42·8	− 3 19	,, 6·0, 9·7	284·8	172·70	·652	2	20 Piscium; p. yel, o.
Arg (Es)	48·0	+40 48	,, 6·5, 8·8	145·4	51·35	1917·847	2	BD + 40° 5167–9; p orge, bl.
h 3223	48·5	− 2 12	,, 8·8, 9·0	355·8	46·00	1918·622	2	
Sh 358	49·8	+31 21	8·0, 9·7	327·0	36·25	·622	2	D>Ang>s.
Es*	54·2	53 19	Fr 8·3, 9·5	105·3	17·19	·655	2	BD+53° 3266.
Es 221	55·6	36 13	,, 8·1, 9·3	234·3	15·00	·655	2	
Du 4	56·2	7 9	,, 9·0, 10·0	263·9	14·97	·655	2	
Arg 47	57·5	59 24	,, 8·5, 9·0	291·0	10·16	·655	2	BD + 59° 2819; Ang <s?
Σ 3056	59·5	33 43	7·4, 9·0	359·8	23·17 AC	·655	2	yel, bl; Ang and D<s

Mr. Frederick J. Hanbury's Observatory:
Brockhurst, East Grinstead.

* Unnumbered.